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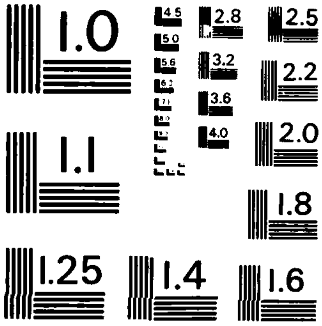
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ADAPTIVE DECENTRALIZED CONTROL

ANNUAL REPORT

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This annual report summarizes some preliminary results of a research effort directed towards the development of adaptive decentralized control systems. The adaptive controller in such a system must operate in the presence of unmodeled dynamics. An input-output approach is presented for analyzing the global stability and robustness properties of adaptive controllers under such circumstances. Conditions for guaranteeing global stability of the error system associated with the adaptive controller, and ensuring boundedness of the adaptive gains, are derived. Specific bounds are presented for certain signals in the control systems.

TABLE OF CONTENTS

I. INTRODUCTION AND SUMMARY

This annual report summarizes work performed on the Adaptive Decentralized Control project (under contract F4920-81-C-0051) during the period June 1982 - July 1983. The objective of this research effort is the development of a new concept for the design of decentralized controllers for large scale systems.

The modeling, analysis and control of large-scale systems is an increasingly important problem in such diverse areas as defense systems, communication and computer networks and transportation systems. The size and complexity of many systems make it difficult or impractical to use centralized control structures. Furthermore, considerations of communication costs, system reliability, computational requirements and response time provide strong incentives for the use of distributed control architectures. The basic focus of our research is on a framework within which decentralized controller structures can be analyzed and developed. The motivation for our proposed approach which we named ADCON (for <u>A</u>daptive <u>D</u>ecentralized <u>CON</u>trol) comes from the following observations about the current status of control theory.

An important aspect of centralized control has been the study of systems with unknown or uncertain (time varying, random) parameters. The investigation of this problem led to an extensive literature on adaptive control (also called: learning or self-organizing systems). The natural progression in developing centralized controllers was from the non-adaptive case to the more difficult problems addressed by adaptive techniques.

The study of decentralized control seems so far to be almost exclusively devoted to non-adaptive techniques. A possible explanation of this state of affairs is the fact that the area of decentralized control of completely known systems still has many unresolved issues and some basic problems are yet to be answered. Under these conditions, there seemed to be little incentive to tackle the more complex adaptive case which deals with partially known systems. However, this line of thinking is based on the experience gained in centralized control and it may be inapplicable in the context of the decentralized problem, which has radically different characteristics. In

fact, adaptive techniques have a central role in decentralized control, which is of a somewhat different nature than the role they play in the centralized problem.

To understand the interrelation between adaptive and decentralized control, we have to re-examine the basic issues underlying the need for decentralized control strategies. The main motivation for considering such strategies arises in the context of complex, large-scale systems where a centralized controller usually requires excessive computational requirements and excessive information gathering networks to make such a controller feasible. In such a system, it is reasonable to assume that the local controller (i.e., the controller of one subsystem in the large system) has only partial information about the rest of the system. Even if the structure of the whole system (i.e., the state equations of all subsystems and their interactions) can be made available to each local controller, the sheer complexity of the problem often limits the usefulness of this information. In fact, attempting to use too much information may be one of the principal stumbling blocks of conventional approaches to decentralized control. Most of these approaches try to solve the (optimal) centralized problem, and then to find clever ways of decentralizing the solution. The shortcomings of this technique and the need for a different point of view are by now widely recognized.

The basic idea underlying our approach is to <u>assume</u> that from the subsystem's point of view, the rest of the system is not exactly known. Thus, the subsystem is aware of its own structure, but it has only an approximate knowledge of the rest of the system, for example, in the form of a reduced order model. (Different subsystems will use different models of the "outside world".) The local controller is then designed on the basis of this partial information. The modeling uncertainty inherent in this procedure makes it necessary to consider robust or adaptive control structures. Note that the uncertainty here is due to the complexity of the system rather than to lack of knowledge or to random effects, which are the traditional sources of uncertainty in centralized control. The idea of replacing a complex deterministic problem by a simple stochastic model is by no means new, and has been used in a variety of physical problems (e.g., statistical

thermodynamics).

The use of reduced order models and partial information greatly simplifies the design and implementation of the decentralized controllers. It raises, however, many difficult questions regarding the conditions under which such a scheme will lead to satisfactory system behavior. What is needed is a theory for the control of interconnected subsystems in the presence of model uncertainties. In an earlier report [36] and in some related papers we made a preliminary study of some of these issues.

An even more difficult set of questions arises with regard to the operation of adaptive controllers in the presence of uncertainty. Currently available adaptive control algorithms have been shown to experience severe difficulties in the presence of unmodeled plant dynamics. We were able to derive conditions which guarantee that the adaptive controller will have specified performance despite plant uncertainty and unmodeled dynamics. These conditions provide guidelines for the analysis and design of robust adaptive controllers. A combination of results from robust control and adaptive control theory was used to prove the main theorem. The main theorem was applied to a number of well-known adaptive structures: the direct adaptive controller, an adaptive observer, the indirect adaptive controller, and a general form of the model reference adaptive controller [40]. We believe that this work represents a significant advance in the field of adaptive control.

In the next section we present an input-output approach for analyzing the global stability and robustness properties of adaptive controllers to unmodeled dynamics. The concept of a tuned system is introduced, i.e., the control system that could be obtained if the plant were known. Comparing the adaptive system with the tuned system results in the development of a generic adaptive error system. Passivity theory is used to derive conditions which guarantee global stability of the error system associated with the adaptive controller, and ensure boundedness of the adaptive gains. Specific bounds are presented for certain significant signals in the control systems. Limitations of these global results are discussed, particularly the requirement that a certain operator be strictly positive real (SPR) -- a condition that is unlikely to hold due to unmodeled dynamics. The work summarized in this

section was performed jointly with Dr. Robert L. Kosut, and will be published in the IEEE Transactions on Automatic Control.

In section 3 we briefly describe some ongoing research, which will be reported more fully at the completion of the current project.

II. ROBUST ADAPTIVE CONTROL: CONDITIONS FOR GLOBAL STABILITY

1. INTRODUCTION

1.1 Background

The analysis and design of adaptive control systems has been the subject of extensive research in the past two decades [1]-[10]. Adaptive techniques provide a way of handling plant uncertainty by adjusting the controller parameters on-line to optimize system performance. An alternative method for handling uncertainty is to use a fixed structure controller designed to provide acceptable performance for a specified range of plant behavior. In principle, adaptive controllers can provide improved performance compared to fixed robust controllers, since they are tuned to the uncertain plant. However, adaptive controllers sometimes exhibit undesirable behavior during the tuning or adaptation process. For example, unmodeled dynamics can cause a rapid deterioration in performance and even instability [11],[12]. This problem is not resolved by increasing the order or complexity of the model. Since the model of any dynamic system, by definition, is not the actual system, it can therefore be argued that unmodeled dynamics are always present, ad infinitum.

The main reason for these difficulties with adaptive controllers seems to be that robustness to unmodeled dynamics was not considered as a design criterion in the development of the adaptive control algorithm. The design objective is global stability of the closed-loop system, e.g., [7], [9] and various assumptions on the structure of the plant are required to achieve that objective. In particular, it is necessary to assume that the plant is linear and time invariant (LTI), that the relative degree of the transfer function is known as well as the sign of the high frequency gain. Such requirements are not practical since real plants are often nonlinear and time-varying and can be accurately represented only by high order (sometimes infinite order [13]) complicated models.

The need for robustness to plant uncertainty is not unique to adaptive control. The problem of robustness is ubiquitous in control theory and has been studied in the context of fixed (nonadaptive) control [14]-[17]. These studies rely on the input/output properties of systems, e.g., [18],[19]. The

predominant reason to examine robustness issues in this way is that the characteritics of unmodeled dynamics, such as uncertain model order, are easily represented. Lyapunov theory, on the other hand, is not well suited for this type of uncertainty. Typically, plant uncertainty is characterized by assuming that the plant belongs to a well defined set. For example, a set description of an uncertain LTI plant is to define a "ball" in the frequency domain. The center of the ball is the nominal plant model, and the radius defines the model error. This set model description is one type of a more general set description, referred to as a conic-sector [15]. The uncertainty in the plant induces an uncertainty in the input/output map of the closed-loop system which can, again be characterized by a conic sector. Performance requirements for the control system can be translated into statements on the conic sector which bounds the closed-loop systems, making it possible to check whether a given design meets specifications, and providing guidelines for robust controller design.

In this study we use the input/output approach to analyze the global stability and robustness properties of continuous-time adaptive controllers with respect to unmodeled dynamics (although we consider only continuous-time algorithms, the input-output formalism can be readily extended to the discrete-time case). By global we mean that no specific magnitude constraint (other than boundedness) is placed on any of the external inputs or initial conditons. We develop an adaptive error system of a general form, by comparing the actual adaptive system with a <u>tuned system</u>, i.e., the control system that could be obtained if the plant were known. This error system is similar to the type used in [7],[8] where the tuned system error output is zero, due to the assumption of perfect modeling. By relaxing this assumption we show that the non-zero outputs of the error system are the inputs to a nonlinear feedback error system consisting of the adaptive algorithm and two feedback (interconnection) operators,denoted by H_{ev} and H_{zv} .

An important consequence of this structure is that the <u>existence</u> of solutions (e.g., tuned system performance) is separated from the <u>stabiity</u> analysis (e.g., stability of the nonlinear error system). In general, the adaptation law is passive; consequently, if H_{ev} is strictly positive real (SPR), then application of passivity theory [19]-[21], provides global

L_2-stability of the map from the tuned system output to the actual adaptive system output, even though the adaptive parameters may grow beyond all bounds. We provide other conditions (e.g., H_{zv} stable) to insure the L_∞ boundedness of the adaptive gains. Similar results are developed to insure L_∞-stability of the error system by using an exponentially weighted passivity theory [19]. These results are summarized in Theorems 1A and 1B.

As a by product of the input/output view we also obtain specific bounds on the L_2 and L_∞ norms of significant signals in the adaptive system. The results are summarized in Corollary 1.

The results in Theorem 1 and Corollary 1 are not essentially new (see e.g., [7],[8]), although they do provide some extentions to previous results. The main contribution, however, is the fact that all the results can be obtained from a generic error system and from the application of nonlinear stablity theorems based on input-output properties. As a consequence of this approach, it is to be expected that conditions for robustness will arise in a natural way. Such robustness results are obtained, but unfortunately,they have a limited practical use. The main limitation is that the global theory (Theorem 1) requires that $H_{ev} \varepsilon$ SPR , which in turn places an upper bound on the size of the unmodeled dynamics in the plant. The details are contained in Lemmas 4.1 and 5.2. This bound is quite restrictive and is easily violated by even the most benign model errors, thus, verifying the results obtained in [11], [12]. To over come this limitation, we construct an SPR compensator, based on the scheme proposed in [22] in the context of robust (non-adaptive) control. Although in the adaptive case the supporing arguments are heuristic, an example simulation shows a positive result.

The input/output analysis presented here provides a generic framework within which it is possible to analyze the robustness of adaptive robust controllers. We believe that this framework can be used to develop practical adaptive control algorithms that can be more readily applied to real systems, than the class of algorithms currently in use.

Since this study merges ideas from several areas, it is necessary to introduce a number of definitions and concepts.

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2. SOME PRELIMINARIES

2.1 Notation

The input/output formulation of multivariable systems is the principal view taken throughout this paper and the notation and terminology used is standard (see e.g. [18],[19]). The input and output signals are assumed to be imbedded in either the normed function space

$$L_p^n = \{x : [0,\infty) \to R^n \mid \|x\|_p < \infty\} \tag{2.1a}$$

or its extention

$$L_{pe}^n = \{x : [0,T] \to R^n \mid \|x\|_{Tp} < \infty, \quad T < \infty\} \tag{2.1b}$$

The respective norms $\|\cdot\|_p$ and $\|\cdot\|_{Tp}$ are defined as follows:

$$\|x\|_p = \lim_{T\to\infty} \|x\|_{Tp} \tag{2.2a}$$

with

$$\|x\|_{Tp} = \begin{cases} \left(\int_0^T |x(t)|^p dt\right)^{1/p}, & p \in [1,\infty) \\ \sup_{t\in[0,T]} |x(t)|, & p = \infty \end{cases} \tag{2.2b}$$

where $|\cdot|$ is the Euclidean norm on R^n. Hence, L_{2e}^n is an inner product space, with inner product $\langle x,y\rangle_T$ of elements $x, y \in L_{2e}^n$ defined by

$$\langle x,y\rangle_T = \int_0^T x(t)'y(t)dt \tag{2.3}$$

and so $\|x\|_{T2} = (\langle x,x\rangle_T)^{1/2}$. If $T \to \infty$ then L_2^n is an inner-product space with inner product $\langle x,y\rangle = \lim_{T\to\infty}\langle x,y\rangle_T$.

2.2 Stability

Systems considered in this paper are described by input/output equations of the form $y = Gu$ where $G: L_{pe}^m \to L_{pe}^n$ is a causal map from u into y, also denoted $u \to y$. The system G is said to be L_p-stable (or simply stable) if G maps $u \in L_p^m$ into $y \in L_p^n$ and if there exists finite constants k and b such that $||Gu||_{Tp} < k\, ||u||_{Tp} + b$, for all $T > 0$ and all $u \in L_{pe}^m$. The smallest k that can be found is referred to as the L_p-gain (or simply gain) of G, denoted $\gamma_p(G)$.

Because we often encounter LTI systems it is convenient to introduce the following notation. Let $R(s)$ and $R_0(s)$ denote the proper and strictly proper rational functions, respectively. Let S and S_0 denote functions in R(s) and $R_0(s)$, respectively, whose poles all have negative real parts. Thus, S and S_0 are the stable, lumped, LTI systems. Denote multivariable systems with transfer function matrices, by $R(s)^{nxm}$, S^{nxm}, etc. For example, $G \in S_0^{nxm}$ means that <u>all</u> elements of G belong to S_0, and so on.

If $G \in S^{nxm}$ then the following L_p-gains are obtained,

$$\gamma_1(G) < \gamma_\infty(G) = \int_0^\infty \bar{\sigma}[G(t)]dt \tag{2.4}$$

$$\gamma_2(G) = \sup_{\omega \in R} \bar{\sigma}[G(j\omega)] \tag{2.5}$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of the matrix A, defined as the positive square root of the maximum eigenvalue of A*A, where * is the conjugate transpose of A. In (2.4), (2.5) G is the operator, $G(j\omega)$ the transfer function matrix, and G(t) is the impulse response matrix.

2.3 Passivity

The following definitions follow those in [19],[21]. Let $G: L_{1e}^m \to L_{1e}^m$ and let μ, ρ be constants with $\mu > 0$. Then, $\forall\, u \in L_{2e}^m$:

G is passive if,

$$< u, G\, u>_T \geq \rho \tag{2.6}$$

G is input strictly passive if,

$$< u, Gu >_T \geq \rho + \mu \| u \|_{T2} \tag{2.7a}$$

G is output strictly passive if,

$$< u, Gu >_T \geq \rho + \mu \| Gu \|_{T2} \tag{2.7b}$$

(μ and ρ are not the same throughout). When $G \in S^{mxm}$ satisfies (2.7), G is said to be strictly positive real (SPR), denoted $G \in SPR^m$. Because SPR systems play a crucial role in the proof of stability of adaptive systems, we introduce the following subsets:

$$SPR_+^m = \{G \in S^{mxm} | \underline{\lambda}(\tfrac{1}{2}[G(j\omega) + G(-j\omega)'] - \mu I) > 0, \forall \omega \in R\} \tag{2.8a}$$

$$SPR_0^m = \{G \in S_0^{mxm} | \underline{\lambda}(\tfrac{1}{2}[G(j\omega) + G(-j\omega)'] - \mu\, G(-j\omega)' G(j\omega)) > 0, \forall \omega \in R\} \tag{2.8b}$$

where $\underline{\lambda}(A)$ denotes the smallest eigenvalue of A. Thus, whenever $G \in S^{mxm}$, conditions (2.7) can be tested in the frequency domain. Moreover, SPR_0^m and SPR_+^m, respectively, separate the strictly proper SPR functions from the proper, but not strictly proper, SPR functions. In the scalar case, the frequency domain conditions simplify because $\underline{\lambda}[G(j\omega) + G(-j\omega)'] = 2\, Re[G(j\omega)]$.

Certain unstable systems in $R(s)^{mxm}$ can be passive by virtue of (2.6). In particular, $G \in R(s)^{mxm}$ is passive if $G(s)$ is positive real. The transfer function matrix $G(s)$ is positive real if: (i) it has no poles in $Re(s) > 0$, (ii) poles on the $j\omega$ axis are simple with a non-negative residue, and (iii) for any $\omega \in R$ not a pole of $G(j\omega) + \underline{G}(-j\omega)' \geq$.

2.4 Model Error

The cornerstone of robust control design is a quantifiable bound on the error between the model used for control design and the actual plant to be controlled. In the adaptive control case considered here the model is a parametric model, where the parameters are not known exactly. The structure of the parametric model can be obtained analytically from physical laws, but this invariably results in a complicated model. Often a simple structure is selected because it is more convenient for analysis and synthesis.

Let P denote the plant to be controlled. In the broadest sense P is a <u>relation</u> in $L_{1e}^m \times L_{1e}^n$, i.e., the set of all possible ordered pairs $(u,y) \varepsilon L_{1e}^m \times L_{1e}^n$ of inputs $u \varepsilon L_{1e}^m$ and outputs $y \varepsilon L_{1e}^n$ that could be generated by the plant [18]. The uncertainty in the plant is denoted by $(u,y) \varepsilon P$.

Let $P_\alpha : L_{pe}^m \rightarrow L_{pe}^n$ denote a parametric model of the plant P with parameters $\alpha \varepsilon R^k$. The parameters can be selected so as to minimize any discrepancies between the model and the plant, i.e.,

$$\inf_{\alpha \varepsilon R^k} \| y - P_\alpha u \|_{Tp} = \| y - P_* u \|_{Tp} \tag{2.9}$$

We will refer to $\alpha_* \varepsilon R^k$ as the <u>tuned model parameters</u> and to $P_{\alpha_*} = P_*$ as the <u>tuned parametric model</u> of the plant. In general, P_* is dependent on the input/output sequence.

Most of the previous work on adaptive control deals with the case where for every $(u,y) \varepsilon P$ there exists a tuned parametric model P_*, such that $P_* = P$. In this paper we consider the presence of unmodeled dynamics, thus, the uncertain plant P cannot be perfectly modeled by any parametric model P_α. Since we will deal exclusively with LTI plants $P \varepsilon R(s)^{n \times m}$, it is convenient to describe this model error in the frequency-domain. Let $B_S(r)$ denote a "ball" in S of radius r, defined by

$$B_S(r) := \{ G \varepsilon S^{n \times m} | \ \bar{\sigma}[G(j\omega)] < r(\omega), \ \omega \varepsilon R \} \tag{2.10}$$

Let the plant to be controlled be described by

$$P = (I + \Delta)P_{*} \tag{2.11a}$$

where $P \in R(s)^{nxm}$ is the plant, $P_{*} \in R(s)^{nxm}$ is the tuned parametric model, and $\Delta \in S^{nxn}$ denotes the unmodeled dynamics. Further, the only knowledge available about Δ is that it is bounded such that

$$\Delta \in B_S(\delta) \tag{2.11b}$$

where $\delta(\omega)$ is known for all frequencies. In other words, while the operator Δ is not precisely known, we do know a bound on its effect. This model description (2.2) is used throughout the paper to precisely define the plant to be controlled in an adaptive system. Following Doyle and Stein [16] we will refer to (2.11b) as an unstructred uncertainty. Note that although Δ is stable, P and P* need not be stable. Hence, the parametric model is implicitly required to capture all unstable poles of the plant. Although this is not severly restrictive - at least on practical grounds - nonetheless, it can be eliminated by definng model error as (stable) deviations in (stable) coprime factors of the plant [23]. As the subsequent analysis is not substantially effected by this choice, we will remain with (2.11) for purposes of illustration.

2.5 Persistent Excitation

From [31], a regulated function $F(\cdot) = R_+ \to R^{nxm}$ is persistently exciting, denoted $F \in PE$, if there exists finite positive constants α_1, α_2, and α_3 such that

$$\alpha_2 I_n > \int_s^{s+\alpha_3} F(t)F(t)'dt > \alpha_1 I_n , \quad \forall s \in R_+ \tag{2.12}$$

The usefulness of a persistently exciting signal is in establishing the exponential stability of the following differential equation which arises in many adaptive and identification schemes, i.e.,

$$\dot{x} = -BFHF'x + w , \qquad x(0) \in R^n \tag{2.13}$$

It is shown in [31] that if $B \in R^{nxm}$, $B = B' > 0$, $H \in SPR_0^m$ or SPR_+^m, and $F \in PE$, then $(w, x(0)) \longmapsto x$ is exponentially stable, i.e., $\exists$ m, $\lambda > 0$ such that

$$|x(t)| \le me^{-\lambda t} |x(0)| + \int_0^t me^{-\lambda(t-\tau)} |w(\tau)| d\tau . \tag{2.14}$$

We will utilize this latter result in section IV in our proof of stablity of the adaptive system.

3. ADAPTIVE ERROR MODEL

In this section we develop a generic adaptive error model which will be used in the subsequent analysis. This requires defining the notions of robust control and tuned control.

Robust and Tuned Control

Consider, for example, the model reference adaptive control (MRAC) depicted in Figure 3.1, consisting of the uncertain plant P, a reference model H_r, and an adaptive controller $C(\hat{\theta})$, where $\hat{\theta}$ is the adaptive gain vector, r is a reference input, d is a disturbance process, and n is sensor noise. Denote by $H(\hat{\theta})$ the closed-loop system relating the external inputs $w = (r', d', n')'$ to the output error e, as depicted in Figure 3.2.. Also, let $w \varepsilon W$ denote the admissable class of input signals.

The objective of the adaptive controller is twofold: (1) adjust $\hat{\theta}$ to a constant $\theta_* \varepsilon R^k$ such that $H(\theta_*)$ has desireable properties; and (2) during adaptation, as $\hat{\theta}$ is adjusted, the error is well behaved. In the usual formulations [7] only (1) is considered and further it is assumed that there exists a matched gain, denoted by $\bar{\theta} \varepsilon R^k$, such that

$$H(\bar{\theta}) = 0 \tag{3.1}$$

The presence of uncertain unmodeled dynamics in the plant eliminate the chance of satisfying the matching condition. Thus, it is more appropriate to define a tuned gain, denoted by $\theta_* \varepsilon R^k$, corresponding to each $(u,y,w) \varepsilon P \times W$, such that

$$H(\theta_*)w < H(\theta)w \quad , \ \forall\, \theta \varepsilon R^k \tag{3.2}$$

The error signal $e_* := H(\theta_*)w$ is referred to as the tuned error. Note that each $(u,y,w) \varepsilon P \times W$ engenders a possibly different θ_*. Also, it is important to distinguish the tuned gain θ_*, from the robust gain $\theta_o \varepsilon R^k$, where

$$\sup_{P \times W} H(\theta_o)w < \sup_{P \times W} H(\theta)w \ , \ \forall\, \theta \varepsilon R^k \tag{3.3}$$

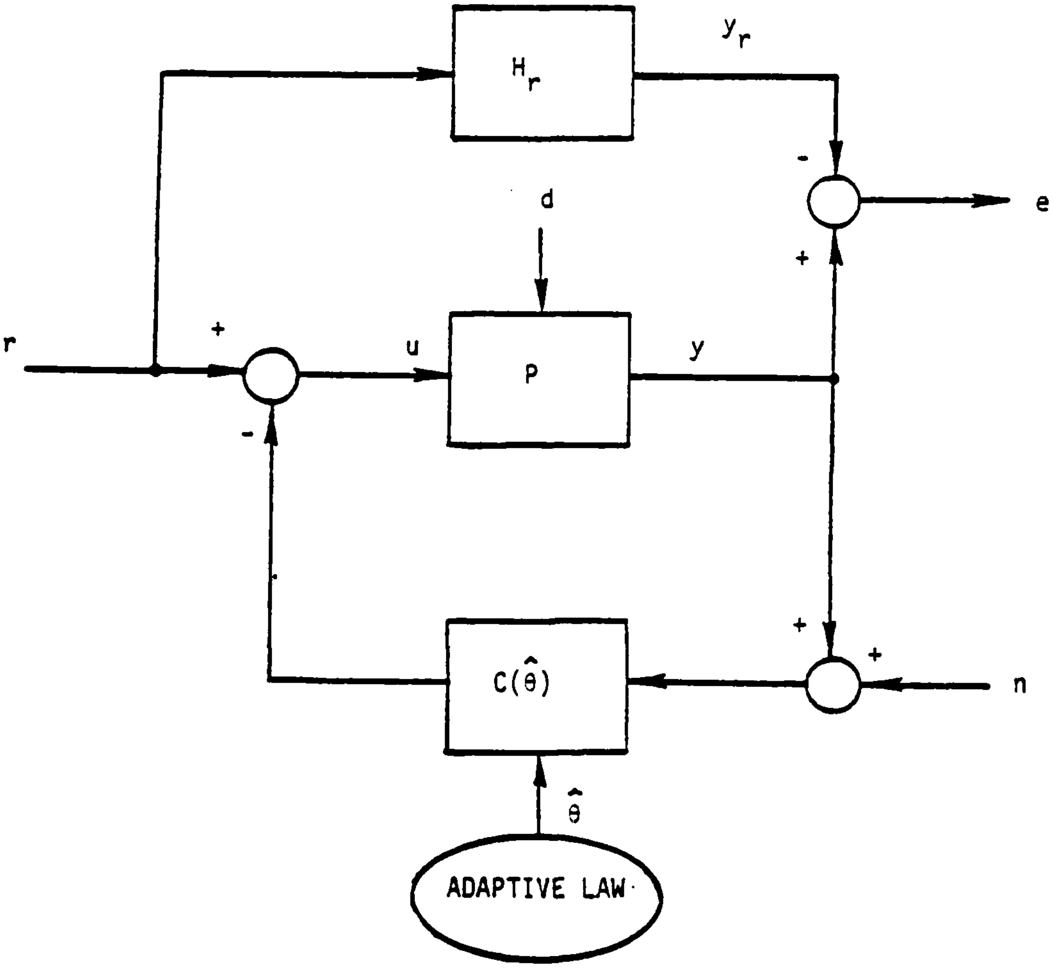

Figure 3.1 A Model Reference Adaptive Controller

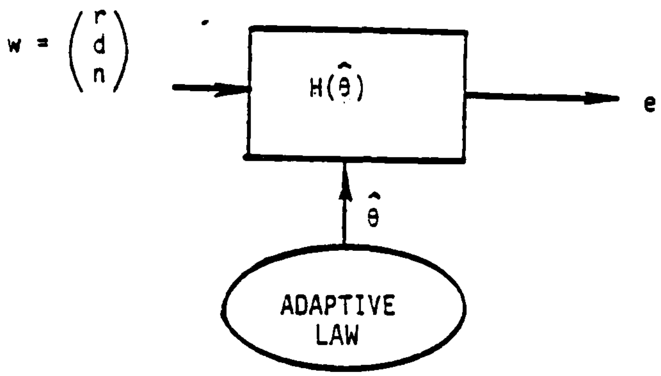

Figure 3.2 Closed-Loop System

The error signal $e_0 := H(\theta_0)w$ is referred to as the robust error. It follows from these definitions that the tuned error is always smaller in norm than the robust error, thus $\forall\ w \in W$,

$$e_* = H(\theta_*)w < e_0 = H(\theta_0)w , \tag{3.4}$$

The tuned controller is, unfortunately, unrealizable since it requires prior knowledge of the actual system $H(\theta)$ (or equivalently, the plant P) and the input w. A practical adaptive controller is likely to have a larger error norm.

Structure of the Adaptive Control

In summary, we consider the multivariable adaptive system, shown in Figure 3.2, and described by

$$e = H(\hat{\theta})w . \tag{3.5}$$

where $e(t) \in R^m$ is the error signal to be controlled, $w(t) \in R^q$ is the external input restricted to some set W, and $\hat{\theta}(t) \in R^k$ is the adaptive gain. The class of adaptive controllers considered here are such that the adaptive gains multiply elements of internal signals $z(t) \in R^k$, referred to as the regressor, to produce the adaptive control signals,

$$f_i = \hat{\theta}_i' z_i \quad , \quad i \in [1,m] \tag{3.6}$$

where $\hat{\theta}_i$ and z_i are k_i-dimensional subsets of the elements in $\hat{\theta}$ and z, respectively. Thus,

$$k = \sum_{i=1}^{m} k_i \tag{3.7}$$

Define the adaptive gain error,

$$\theta(t) := \hat{\theta}(t) - \theta_* \tag{3.8}$$

where $\theta_* \in R^k$ is the tuned gain (3.4). Also, define the adaptive control error signals,

$$v_i := \theta_i' z_i \quad , \quad i = 1, \ldots, m \tag{3.9}$$

An equivalent expression is,

$$v = Z'\theta \tag{3.10a}$$

where the time-varying matrix Z is defined by

$$Z = \text{block diag}(z_1, z_2, \ldots, z_m) \tag{3.10b}$$

To describe the relations among the signals e, z, v, and w we introduce the <u>interconnection system</u> $H_I : (w,v) \to (e,z)$, as shown in Figure 3.3. In particular, let $H_I \in R(s)^{(m+k)x(m+q)}$, and where H_I is defined by,

$$\begin{pmatrix} e \\ z \end{pmatrix} := H_I \begin{pmatrix} w \\ v \end{pmatrix} := \begin{pmatrix} H_{ew} \\ H_{zw} \end{pmatrix} \begin{pmatrix} -H_{ev} & w \\ -H_{zv} & v \end{pmatrix} \tag{3.11}$$

In effect, this structure serves to isolate the adaptive control error v, from the rest of the system. When the adpative control is tuned, $\theta = 0$ and $v = 0$; consequently, the tuned error signal (3.4) is,

$$e_* := H(\theta_*)w = H_{ew}w \tag{3.12}$$

We can also define a tuned regressor signal,

$$z_* := H_{zw}w \tag{3.13}$$

In general, all the subsystems in H_I are dependent on the tuned gains θ_* .

The interconnection system can also be written as,

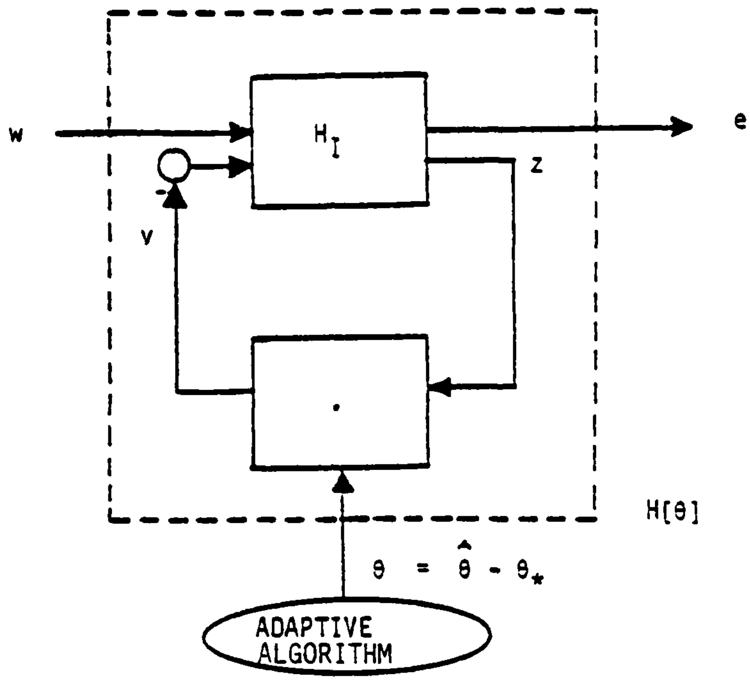

Figure 3.3 Interconnection Structure

$$e = e_* - H_{ev}v \tag{3.14a}$$

$$z = z_* - H_{zv}\,v \tag{3.14b}$$

with v given by (3.10). To complete the error model requires describing the <u>adaptative algorithm</u>, i.e., the means by which $\hat{\theta}(t)$ is generated. We will consider two typical algorithms. A constant gain (gradient) algorithm [7]:

$$\dot{\hat{\theta}} = \Gamma\, Z\, e \tag{3.15}$$

where $\Gamma \in R^{k\times k}$, $\Gamma = \Gamma' > 0$, and a similar but nonlinear gain algorithm:

$$\dot{\hat{\theta}} = \Gamma(Ze - \rho(\hat{\theta})\hat{\theta}) \tag{3.16a}$$

where $\rho : R^k \to R_+$ is a <u>retardation function</u>, whose purpose is to prevent $\hat{\theta}$ from growing too quickly in certain situations. Although many functions will suffice we will select the one proposed in [24], namely:

$$\rho(\hat{\theta}) := \begin{cases} (\|\hat{\theta}\|/c - 1)^2, & \|\hat{\theta}\| > c := \max\|\theta_*\| \\ 0 & , \|\hat{\theta}\| \leq c \end{cases} \tag{3.16b}$$

The complete adaptive error system, is shown in Figure 3.4. Note that the error system is composed of two subsystems: a linear subsystem Σ_L and a non-linear subsystem Σ_N .

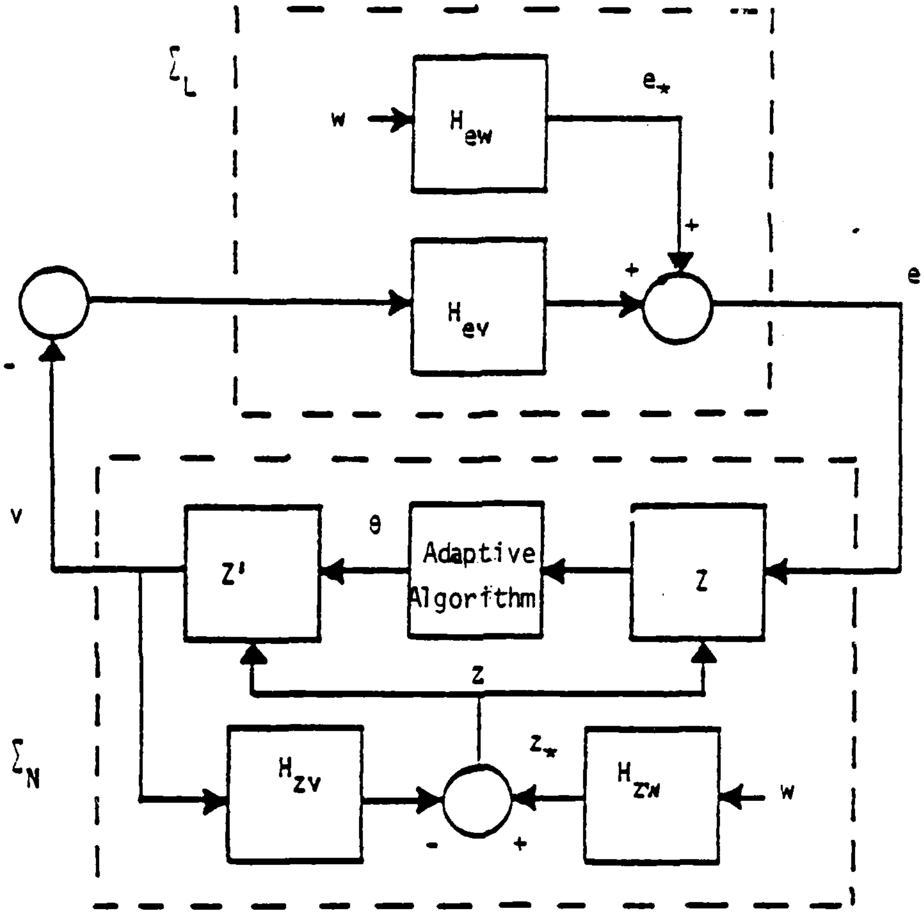

Figure 3.4 Adaptive Error System

4. CONDITIONS FOR GLOBAL STABILITY

The theorems stated below give conditions for which the adaptive error system (Fig. 3.4) is guaranteed to have certain stability and performance properties. Proofs are given in Appendix A. Heuristically, however, the basis for the proofs is application of the Passivity Theorem ([19], pg. 182). It turns out that the map $e \to v$ is passive. Thus, if H_{ev} is SPR^m , then the map $e_* \to (e,v)$ is L_2-stable even though z and/or θ can grow without bounds. Further restrictions, provided below, cause θ and z to be bounded. (We use the notation "$x \to 0$ (exp.)" to mean that $x(t) \to 0$ (exponentially) as $t \to \infty$.)

Theorem A: Global Stability

For the adaptive error system shown in Figure 3.4, assume that:

(A1) The system is well-posed in the sense that all inputs $w \varepsilon W$ produce signals $e.v,z, \theta$, and $\dot{\theta}$ in $L_{\infty e}$.

$$(4.1a)$$

(A2) $H_{zv} \varepsilon S_0^{kxm}$ (4.1b)

(A3) $H_{ev} \varepsilon SPR_+^m$ (4.1c)

Under these conditions:

(i) If $(e_*, \dot{e}_*) \varepsilon L_2^m \cap L_\infty^m (\Longrightarrow e_* \to 0)$ and $(z_*, \dot{z}_*) \varepsilon L_\infty^k$ then with algorithm (3.15) or (3.16):

(i-a) $(\theta,\ddot{\theta}) \varepsilon L_\infty^k$, $\dot{\theta} \varepsilon L_2^k \cap L_\infty^k$, and $\dot{\theta} \to 0$. (4.a)

(i-b) $e \varepsilon L_2^m \cap L_\infty^m$; $\dot{e} \varepsilon L_\infty^m$, and $e-e_* \to 0$. (4.2b)

(i-c) $v \varepsilon L_2^m \cap L_\infty^m$, $\dot{v} \varepsilon L_\infty^m$, and $v \to 0$. (4.2c)

(i-d) $(z,\dot{z}) \in L_\infty^k$, $(z-z_*, \dot{z}-\dot{z}_*) \in L_2^k \cap L_\infty^k$, and $z-z_* \to 0$ exp. (4.2d)

(i-e) If, in addition, $e_* = 0$ (matched) and $z_* \in PE$ then
$(\theta, \dot{\theta}, e-e_*, v, z-z_*) \to 0$ exp.
(4.2e)

(ii) If $(e_*, \dot{e}_*) \in L_\infty^m$ and $(z_*, \dot{z}_*) \in L_\infty^k$, then with algorithm (3.15):

(ii-a) $z \in L_\infty^k$ (4.3)

(ii-b) With the addition of either algorithm (3.16) or $z \in PE$ it follows that the elements of $\theta, \dot{\theta}, e, \dot{e}, v, \dot{v}$, and $\dot{z}$ are in L_∞ .
(4.4)

Theorem 1B: Global Stability

Replace (A3) in Theorem 1 by

(A3)' $H_{ev} \in SPR_0^m$ (4.5)

(i) If $(e_*, \dot{e}_*) \in L_2^m \cap L_\infty^m$ ($\Rightarrow e_* \to 0$) , and $(z_*, \dot{z}_*) \in L_\infty^k$ then with algorithm (3.15) or (3.16)

(i-a) $(\theta, \ddot{\theta}) \in L_\infty^k$, $\dot{\theta} \in L_2^k \cap L_\infty^k$, $\dot{\theta} \to 0$ (4.6a)

(i-b) $e \in L_2^m \cap L_\infty^m$, $\dot{e} \in L_\infty^m$, $e - e_* \to 0$ (4.6b)

(i-c) $(v,\dot{v}) \in L_\infty^m$ (4.6c)

(i-d) $(z,\dot{z}) \in L_\infty^k$, $(z-z_*, \dot{z}-\dot{z}_*) \in L_2^k \cap L_\infty^k$,
and $z-z_* \to 0$. (4.6d)

(i-e) If, in addition, $e_* = 0$ (matched) and $z_* \in PE$,
then $(\theta, v) \to 0$ exp. (4.6e)

(ii) If $(e_*, \dot{e}_*) \varepsilon L_\infty^m$ and $(z_*, \dot{z}_*) \varepsilon L_\infty^k$, then with algorithm (3.15):

(ii-a) $z \varepsilon L_\infty^k$ (4.7d)

(ii-b) With the addition of either $z \varepsilon PE$ or algorithm (3.16), the elements of $\theta, \dot{\theta}, e, \dot{e}, v, \dot{v}$, and $\dot{z}$ are in L_∞. (4.7b)

Corollary 1: Performance Bounds

Suppose z_* and e_* satisfy the conditions in (i) of Theroems 1A or 1B.

(i) Let $H_{ev} \varepsilon SPR_+^m$, i.e., $\exists\, \mu, \gamma > 0$ such that $\forall \omega \varepsilon R$,

$$\bar{\sigma}[H_{ev}(j\omega)] < \gamma \text{ and } \tfrac{1}{2}[H_{ev}(j\omega) + H_{ev}(-j\omega)'] > \mu\, I_m \tag{4.8a}$$

Then, bounds on $\|e\|_2$ and $\|\theta\|_\infty$ can be obtained from:

$$\|e-e_*\|_2 < \frac{\gamma}{2\mu}\left[\|e_*\|_2 + \left(\|e_*\|_2^2 + 2\mu\, \theta(0)'\, \Gamma^{-1}\theta(0)\right)^{1/2}\right] \tag{4.8b}$$

$$\|\theta'\Gamma^{-1}\theta\|_\infty < \theta(0)'\, \Gamma^{-1}\theta(0) + 2\|e\|_2\|e-e_*\|_2/\gamma \tag{4.8c}$$

(ii) Let $H_{ev} \varepsilon SPR_0^m$, i.e., $\exists\, \mu, q, k > 0$ such that $\forall\, \omega \varepsilon R$,

$$\tfrac{1}{2}[H_{ev}(j\omega) + H_{ev}(-j\omega)'] > \mu\, H_{ev}(-j\omega)'\, H_{ev}(j\omega) \tag{4.9a}$$

$$\tfrac{1}{2}[G_{ev}(j\omega) + G_{ev}(-j\omega)'] > k\, I_m \tag{4.9b}$$

$$G_{ev}(s) := (1 + qs)\, H_{ev}(s) \tag{4.9c}$$

Then, bounds on $\|e_2\|$ and $\|\theta\|_\infty$ can be obtained from:

$$\|e\|_2 < \frac{1}{2\mu k}\left[\|e_*+q\dot{e}_*\| + \left(\|e_*+q\dot{e}_*\|_2^2 + 2k^2\mu\theta(0)'\Gamma^{-1}\theta(0)\right)^{1/2}\right] \tag{4.9d}$$

$$\|\theta'\Gamma^{-1}\theta\|_\infty < \theta(0)'\Gamma^{-1}\theta(0) + \frac{1}{k}\|e_*+q\dot{e}_*\|_2\|e\|_2 \tag{4.9c}$$

Discussion

(1) Theorems 1A and 1B give conditions under which the adaptive error system is globally stable. Essentially, conditions are imposed on the interconnection subsystems in H_I. In particular, $H_{ev} \in SPR^m$ and $H_{zv} \in S_0^{k \times m}$ are direct requirements, whereas the restrictions on the tuned signals e_* and z_*, indirectly impose requirements on H_{ew} and H_{zw}. These latter requirements are dependent on knowledge about $w \in W$. For example, if w is a constant, then the assumption that $e_* \to 0$ (Theorem 1A-i) requires that the tuned feedback system is a Type-I robust servomechanism, i.e., the transfer junction $H_{ew}(0) = 0$ for all $(u,y) \in P$.

(2) Corollary 1 gives explicit bounds on signals in the error system. These bounds can be used to evaluate the adaptive system design. Moreover, the bounds allow a coarse determination as to the efficacy of adaptive control vs. robust control. By comparing, for example, the adaptive error $\|e\|_2$ from (4.8) with the robust error $\|e_0\|_2$ from (1.5), it is possible to obtain a quantifiable measure of performance degradation during adaptation.

(3) Although Theorems 1A and 1B are essentially the same, there are slight difference worth noting. These differences arise because in 1A, $H_{ev} \in SPR_+^m \Rightarrow H_{ev}(s)$ is proper but not strictly proper, whereas in 1B, $H_{ev} \in SPR_0^m \Rightarrow H_{ev}(s)$ is strictly proper. Thus, comparing part (i) in 1A and 1B, we see that in 1B, $v, \dot{v} \in L_\infty^m$ whereas in 1A, v is additonally in L_2^m and $v \to 0$.

(4) The use of persistent excitation or gain retardation is seen in part (ii) of theorems 1A and 1B to provide the means to guaranty bounded signals. Other schemes based on signal normalizations or dead-zones can provide similar results, e.g. [32],[33]. The effect of these conditions is to provide an L_∞-stability which is not present otherwise. The persistent excitation condition actually supplies exponential stability, which is stronger than L_∞-stability, as provided, for example, by the gain retardation (see proof in Appendix A).

(5) The persistent excitation requirements in parts (i) and parts (ii)

are different. In parts (i), $z_* \varepsilon PE$, whereas in parts (ii), $z \varepsilon PE$. The different assumptions arise because in parts (i) we enforce the matched condition $e_*=0$. Hence, $z_* \varepsilon PE \Rightarrow z \varepsilon PE$. This follows from (i-d) where $z - z_* \rightarrow 0$ expoentially. Also, with $e_* = 0$, a bounded disturbance added to the reference can cause $z \varepsilon PE$ without forcing, $e_* \varepsilon L_\infty$. In parts (ii), which is more realistic, we disallow the matched condition, and hence, $e_* \varepsilon L_\infty$. Thus, $z \varepsilon PE$ is the weakest assumption to make. However, since z is inside the adaptive loop, it is very different to guarantee $z \varepsilon PE$ by injecting external signals. Note also (in both parts(ii)) that without retardation or PE it is possible for the regressor to remain bounded even though the adaptive parameters may grow unbounded. Similar results have been reported elsewhere, e.g. [24].

Robustness to Unmodeled Dynamics

Since the theorems impose requriements on the input/output properties of the interconnection system, it follows that the effect of model error on these properties determines the stability robustness of the adaptive system. For example, both theorems require that $H_{ev} \varepsilon SPR^m$. Suppose, however, that H_{ev} has the form,

$$H_{ev} = (I + \tilde{H}_{ev})\bar{H}_{ev} \tag{4.10}$$

where $\tilde{H}_{ev}$ is the projection onto H_{ev} of the plant uncertainy operator Δ ; and $\bar{H}_{ev}$ is the nominal transfer function when there is no uncertainty, i.e., when $\Delta = 0$. Thus, $\bar{H}_{ev}$ is a function of the tuned parametric model P_* and the tuned controller gains θ_* . (See Section V for more specific formulae, e.g. (5.5).)

Conditions to insure that $H_{ev} \varepsilon SPR_+^m$ despite uncertainty in H_{ev} is provided by the following:

Lemma 4.1: Let H_{ev} be given by (4.3). Then $H_{ev} \varepsilon SPR_+^m$ if the following conditions hold:

(i) $\bar{H}_{ev} \varepsilon SPR_+^m$ (4.11a)

(ii) $\tilde{H}_{ev} \varepsilon B_S(k)$ where $\forall\ \omega \varepsilon R$, (4.11b)

$$k(\omega) < \frac{1}{2} \underline{\lambda}[\bar{H}_{ev}(j\omega) + \bar{H}_{ev}(-j\omega)']/\bar{\sigma}[\bar{H}_{ev}(j\omega)] \tag{4.11c}$$

<u>Proof:</u> Define $\underline{\mu}(\cdot)$: $C^{mxm} \rightarrow R$ by

$$\underline{\mu}\ (A) = \frac{1}{2} \underline{\lambda}(A+A^*)$$

where * denotes conjugate transpose. Then, using definition (2.8) with (4.10) - (4.11) we obtain

$$\underline{\mu}[H_{ev}(j\omega)] \doteq \underline{\mu}[\bar{H}_{ev}(j\omega) + \tilde{H}_{ev}(j\omega)\bar{H}_{ev}(j\omega)]$$

$$> \underline{\mu}[\bar{H}_{ev}(j\omega)] - \bar{\sigma}(\tilde{H}_{ev}(j\omega)]\bar{\sigma}[\bar{H}_{ev}(j\omega)] > 0\ .$$

Hnece, $H_{ev} \varepsilon\ SPR_+^m$.

<u>Comments</u>

(1) In order to apply Lemma 4.1 it is necessary to have a detailed description of how the plant uncertainty Δ propagates onto the interconnection uncertainty $\tilde{H}_{ev}$. This type of uncertainty propagation was explored in depth by Safonov [25] and more sophisticated expressions then (4.4b) are available to describe the uncertain operator $\tilde{H}_{ev}$. Section 5 contains more detail on this issue.

(2) In the scalar case (4.11c) becomes

$$\begin{aligned} k(\omega) &< Re[\bar{H}_{ev}(j\omega)]/|\bar{H}_{ev}(j\omega)| \\ &= \cos \measuredangle\ [\bar{H}_{ev}(j\omega)] \end{aligned} \tag{4.12}$$

Since $\bar{H}_{ev}\ \varepsilon\ SPR^m$ by assumption, $k(\omega)$ is always positive for $\omega \varepsilon R$; but because of the cosine function, $k(\omega) < 1$. In Section 6 we show that this limitation on the effect of model error is easily violated by even the most benign type of unmodeled dynamics in the plant. Methods which overcome this

limitation are discussed in Section 7. The requirement that $k(\omega) < 1$ also holds for any multivariable $\bar{H}_{ev} \in SPR^m$. To see this let $\bar{H}_{ev}$ have the polar decomposition,

$$\bar{H}_{ev} = G_\ell W_{ev} = W_{ev} G_r \tag{4.13}$$

where G_ℓ, G_r are Hermitian and W_{ev} is unitary. Since $\bar{\sigma}(\bar{H}_{ev}) = \bar{\sigma}(G_\ell) = \bar{\sigma}(G_r)$, it follows that

$$k(\omega) < \bar{\sigma}[W_{ev}(j\omega)] < 1 \tag{4.14}$$

In the case of scalar systems, the condition $k(\omega) < 1$ can be interpreted in terms of a limitation on relative degree of $H_{ev}(s)$. A necessary condition for $H_{ev} \in SPR$ is that the relative degree of $H_{ev}(s)$ does not exceed one i.e., phase limited to ±90°. Rohrs, et al. [12] show that this necessitates precise knowledge of plant order, and hence, is not a feasible requirement in the presence of an unstructured uncertainty (2.12), where the order is unknown. In the multivariable case it is awkward to talk about relative degree or phase, however, (4.14) expresses the same limitation.

(3) In several instances, e.g., [9],[26],[27], it has been reported that the SPR condition has been eliminated. In each case, however, it can be verified that the operator H_{ev} = positive constant, which is SPR. But, these studies do not account for unmodeled dynamics, thus, in the notation of (4.10), only $\bar{H}_{ev}$ = positive constant. Lemma 4.1 then provides the means to evaluate the effect of unmodeled dynamic.

5. APPLICATION TO MODEL REFERENCE ADAPTIVE CONTROL

Consider the model reference adaptive control (MRAC) system, shown in Figure 5.1, consisting of: an uncertain scalar plant $P \in R_0(s)$; a reference model $H_r \in S_0$; and filters with $F \in S_0^{\ell \times 1}$. The plant is affected by a disturbance d and a reference command r. The system equations are:

$$e = y - y_r \tag{5.1a}$$

$$y_r = H_r r \tag{5.1b}$$

$$y = d + Pu \tag{5.1c}$$

$$u = -\hat{\theta}' z = -(\hat{\theta}_1' z_1 + \hat{\theta}_2' z_2) \tag{5.1d}$$

$$z_1 = F u, \; z_2 = F(y-r) \tag{5.1e}$$

Assume that the adaptive law is given by (3.15), thus,

$$\dot{\theta} = \Gamma z e \tag{5.1f}$$

Let the plant uncertainty be described by(2.12), i.e.,

$$\Delta := \frac{P-P_*}{P_*} \in B_S(\delta) \tag{5.1g}$$

where $P_* \in R_0(s)$ is a tuned parametric model for P. Let the filter dynamics be given by

$$F(s) = \left(\frac{1}{L(s)}, \frac{s}{L(s)}, \ldots, \frac{s^{\ell-1}}{L(s)}\right)' \tag{5.1h}$$

where L(s) is a stable monic polynomial of degree ℓ . Thus, $\hat{\theta}_1(t), \hat{\theta}_2(t) \in R^{\ell}$ and so $\hat{\theta}(t) \in R^{2\ell}$. Using the definition of tuned gain (3.2) we get,

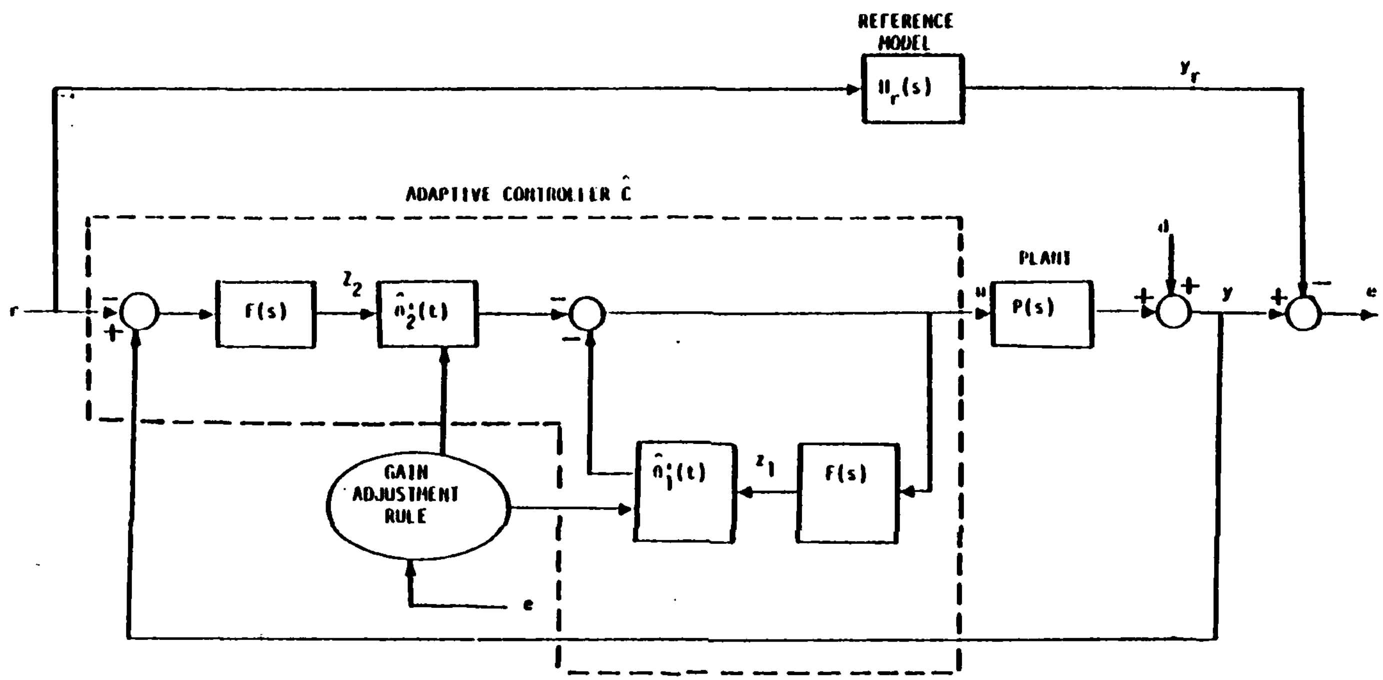

Figure 5.1 MRAC System With Scalar Plant

$$
\begin{aligned}
u &= -\hat{\theta}'z = -(\theta_* + \theta)'z \\
&= -(\theta_{*1}' z_1 + \theta_{*2}' z_2) - v \,, \quad v := \theta' z \quad \text{from (3.6)} \\
&= -\frac{A_{*1}}{L} u + \frac{A_{*2}}{L}(r-y) - v
\end{aligned}
$$

Finally,

$$
u = \frac{A_{*2}/L}{1+A_{*1}/L}(r-y) - \frac{1}{1+A_{*1}/L} v := C_*(r-y) - \frac{1}{1+A_{*1}/L} v \tag{5.2}
$$

where A_* and A_{*2} are polynomials, each of degree $\ell-1$, whose coefficients are the elements of the tuned gains θ_{*1} and θ_{*2}, respectively; and C_* denotes the tuned controller. The tuned system ($\theta=0$) is shown in Figure 5.2.

In terms of the uncertain plant P, the adaptive error system (Fig. 3.4) corresponding to this MRAC system, has tuned signals:

$$
e_* = (1 + PC_*)^{-1} d + [(1+PC_*)^{-1} PC_* - H_r] r \tag{5.3a}
$$

$$
z_* = \begin{bmatrix} F(1+PC_*)^{-1} C_*(r-d) \\ F(1+PC_*)^{-1}(d-r) \end{bmatrix} \tag{5.3b}
$$

and interconnections:

$$
H_{ev} = (1+PC_*)^{-1} P (1+A_{*1}/L)^{-1} \tag{5.3c}
$$

$$
H_{zv} = \begin{bmatrix} F(1+PC_*)^{-1}(1+A_{*1}/L)^{-1} \\ F(1+PC_*)^{-1} P (1+A_{*1}/L)^{-1} \end{bmatrix} \tag{5.3d}
$$

The error system can also be described so as to highlight the model error Δ . The following definitions are convenient:

$$
T_* := (1+P_*C_*)^{-1} P_* C_* := 1 - S_* \tag{5.4a}
$$

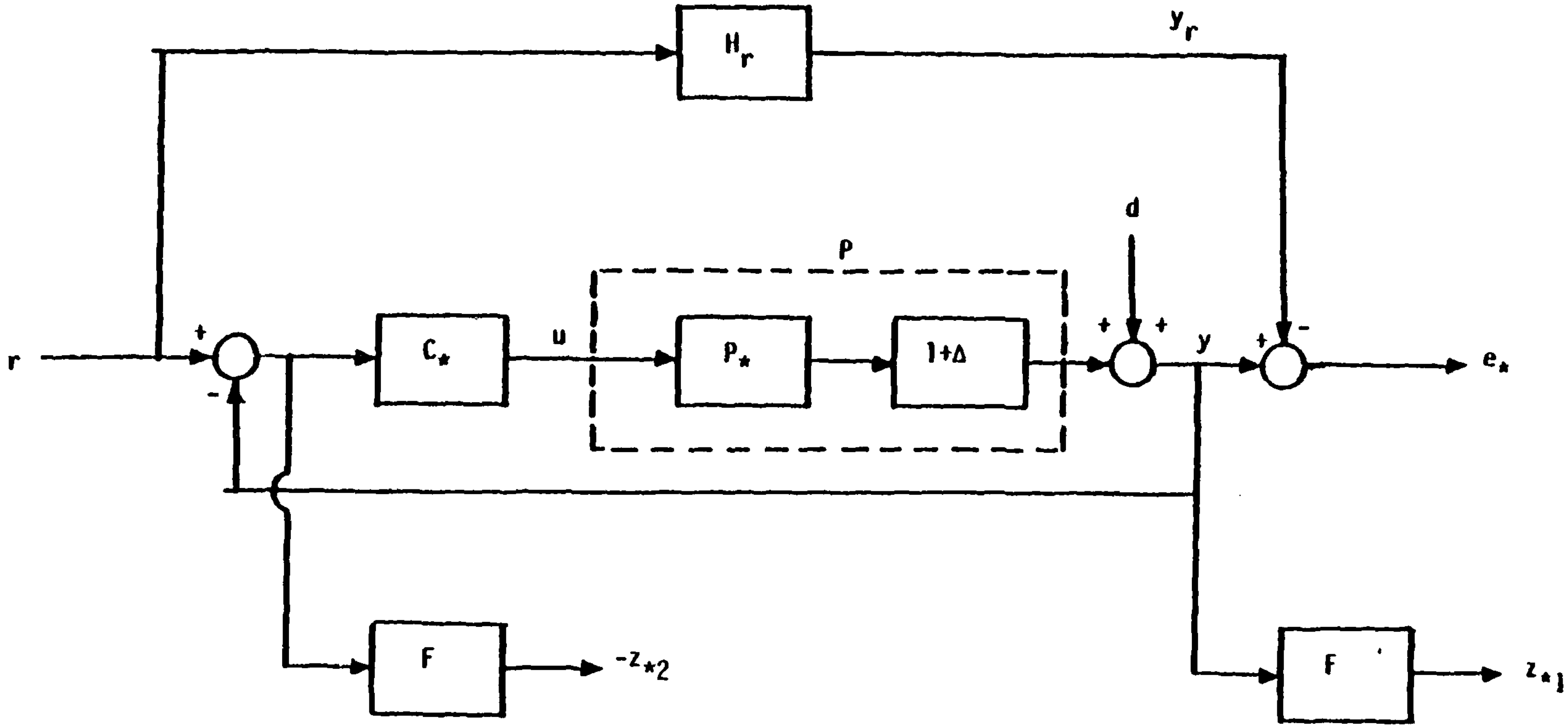

Figure 5.2 Tuned System

$$K_* := H_{ev}\Big|_{\Delta=0} = (1+P_*C_*)^{-1}P_*(1+A_{*1}/L)^{-1} \tag{5.4b}$$

Thus, the error system (5.3) can be also be expressed as:

$$e_* = S_*(1+\Delta T_*)^{-1}d + (T_*(1+\Delta)(1+\Delta T_*)^{-1}-H_r)r \tag{5.5a}$$

$$z_* = \begin{bmatrix} F\ S_*C_*(1+\Delta T_*)^{-1}(r-d) \\ \\ F\ S_*(1+\Delta T_*)^{-1}(d-r) \end{bmatrix} \tag{5.5b}$$

$$H_{ev} = K_*(1+\Delta)(1+\Delta T_*)^{-1} \tag{5.5c}$$

$$H_{zv} = \begin{bmatrix} F\ K_*P_*^{-1}(1+\Delta T_*)^{-1} \\ \\ F\ K_*(1+\Delta)(1+\Delta T_*)^{-1} \end{bmatrix} \tag{5.5d}$$

The result that follows in Lemma 5.1 gives conditions under which $H_{ev} \in SPR_o$ and $H_{zv} \in S_o^{2\ell x1}$, despite model error; thus conditions (A1)-(A3) of Theorems 1A and 2B are satisfied. Additional requirements are necessary to establish the class of tuned signals $e*$ and $z*$ as given by (5.5a) and (5.5b), respectively. These requirements are discussed following Lemma 5.1.

<u>Lemma 5.1:</u> For the adaptive system (5.3) or (5.5) $H_{ev} \in SPR_o$ and $H_{zv} \in S_o^{2\ell x1}$ if the following conditions are all satisfied:

(i) $$P_*(s) = \frac{g(s^{n-1}+\beta_1 s^{n-2}+ \dots + \beta_{n-1})}{s^n+\alpha_1 s^{n-1}+ \dots + \alpha_n} = \frac{gN_*(s)}{D_*(s)} \tag{5.6a}$$

(ii) $N_*(s)$ is a stable monic polynomial (5.6b)

(iii) $g > 0$ (5.6c)

(iv) $K_*(s) = \dfrac{g\ K_1(s)}{K_2(s)} \in SPR_o$ where $K_1(s)$ and $K_2(s)$ are monic stable

polynomials.

(5.6d)

(v) $\ell = \deg L(s) > n + \deg K_1(s) - 1$ (5.6e)

(vi) $\Delta \in B_S(\delta)$ is such that

$$\left.\begin{aligned} \delta(\omega) < \bar{\delta}(\omega) &:= \eta(\omega)[\eta(\omega)|T_*(j\omega)| + |S_*(j\omega)|]^{-1} \\ \eta(\omega) &:= \cos \measuredangle [K_*(j\omega)] \end{aligned}\right\} \forall\, \omega \in R,$$

(5.6f)

Proof: See Appendix B.

Discussion

(1) Condition (i)-(v) of Lemma 5.1 are restatements of known results, but normally they apply to the actual plant P, e.g. [7]. In Lemma 5.1, however, these conditions apply to the parametric model $P*$ -- not to the actual plant. As such, they are easier to satisfy, since the parametric model is somewhat arbitrary. This flexibility is penalized by an increase in model error. For example, if the actual plant has a relative degree of 2, then choosing a parametric model of relative degree 1 -- as required by condition (i) -- incrases the high frequency model error.

(2) Condition (vi) imposes an upper bound $\bar{\delta}$ on the model error associated with the chosen parametric model. This condition simultaneously insures that $H_{ev} \in SPR_0$ despite model error, and that the tuned system is stable (see proof in Appendix B).

(3) It is easily verified that $\bar{\delta}(\omega) < 1$, as was discussed following Lemma 4.1. In fact, even the "optimally tight" bound (see [25] for details on this calculation) given by,

$$\bar{\delta} = \frac{1}{2|T|\eta}\left[-|1-T| + \left(|1+T|^2 + 4\eta\ \mathrm{Re}(KT/|K|\right)^{1/2}\right] \tag{5.7}$$

is also restricted to be less than 1. This limitation severely restricts the type of admissable model error. This issue is pursued in Section 6.

(4) To guarantee global stability using the adaptive law (5.1f), property (i) of Theorem 1 requires that $e_* \to 0$ and $z_*, \dot{z}_* \in L_\infty^{2\ell}$ for all r and d. For example, let r and d be any bounded signals such that $r \to$ constant and $d \to$ constant as $t \to \infty$. Property (i) of Theorem 1 is satisfied if:

$$\delta(0) = 0 \tag{5.8a}$$

$$T_*(0) = H_r(0) = 1 \tag{5.8b}$$

Zero model error at DC (5.8a) is certainly to be expected from even the most crude tuned parametric model.

(5) Let r be bounded such that $r \to$ constant as $t \to \infty$, but let d be just bounded, i.e., $d \in L_\infty$. In this case it is not possible to guarantee $e_* \to 0$, but we can guarantee that $e_* \in L_\infty$. To obtain global stability in this case, requires the introduction of the retardation term (3.16) into the adaptive law (5.1f), see part (ii) of Theorems 1A or 1B.

(6) It is possible to obtain versions of Lemma 5.1 for adaptive systems of different forms, e.g., indirect adaptive [5]. Also, the use of "multipliers", e.g. [4], can be accounted for as well. The multiplier effectively makes use of the availability of $\dot{\hat{\theta}}$ as a signal; and this allows rel deg $(P_*) = 2$ rather than 1 as required by condition (i) of Lemma 5.1.

6. LIMITATIONS IMPOSED BY THE SPR CONDITION

The fact that the model error bound given in condition (vi) of Lemma 5.1 can not exceed one has unfortunate consequences.

Example 1

Consider a plant with transfer function,

$$P(s) = P_*(s)\,\frac{ab}{(s+a)(s+b)} \tag{6.1}$$

where P_* is the parametirc model, with two unmodeled stable poles at -a and -b. Suppose,also, that b is much greater than a, and that a is much greater than the bandwidth of $P_*(s)$. This situation seems benign -- and most likely a certainty. Comparing (6.1) with (5.1g) gives,

$$\delta(\omega) = \omega\left[\frac{\omega^2 + (a+b)^2}{(\omega^2+a^2)(\omega^2+b^2)}\right]^{1/2} > 1$$

for all frequencies $\omega > (ab/2)^{1/2}$, thus, condition (vi) of Lemma 5.1 is violated, and global stability cannot be guaranteed. The following example illustrates this point.

Example 2

Consider the example MRAC system (Fig. 5.1) studied by Rohrs et al. [12], where:

$$P(s) = \frac{2}{s+1}\,\frac{229}{(s+15)^2+4}$$

$$H_R(s) = \frac{3}{s+3}$$

$$u = -\hat{\theta}_1 y + \hat{\theta}_2 r$$

$$\dot{\hat{\theta}}_1 = ye, \ \hat{\theta}_1(0) = .65$$

$$\dot{\hat{\theta}}_2 = -r\ e, \ \hat{\theta}_2(0) = 1.14$$

Let r = constant and $d = 0$. Thus, $e_* \to 0$ exponentially when the tuned gains are such that (5.8) is satisfied, i.e.,

$$T_*(0) = \frac{2\theta_{*2}}{1+2\theta_{*1}} = H_r(0) = 1$$

Even though $(\theta_{*1}, \theta_{*2})$ exist to satisfy this, $H_{ev}(s)$ is not SPR, and so global stability is not guaranteed. Simulation runs with $r = .4$ and $r = 4.0$ are shown in Figures 6.1 and 6.2, respectively. With the small input (Fig. 6.1) we see a stable response which tracks the reference very well. With the large input (Fig. 6.2) the response is still stable, but large oscillations are taking place. Larger inputs will eventually drive the system unstable, e.g. [12].

In this example, if the tuned model is taken to be $P_*(s) = 1/(s+1)$ then it is easily verified that model error $\delta(\omega)$ is greater than one at some frequency.

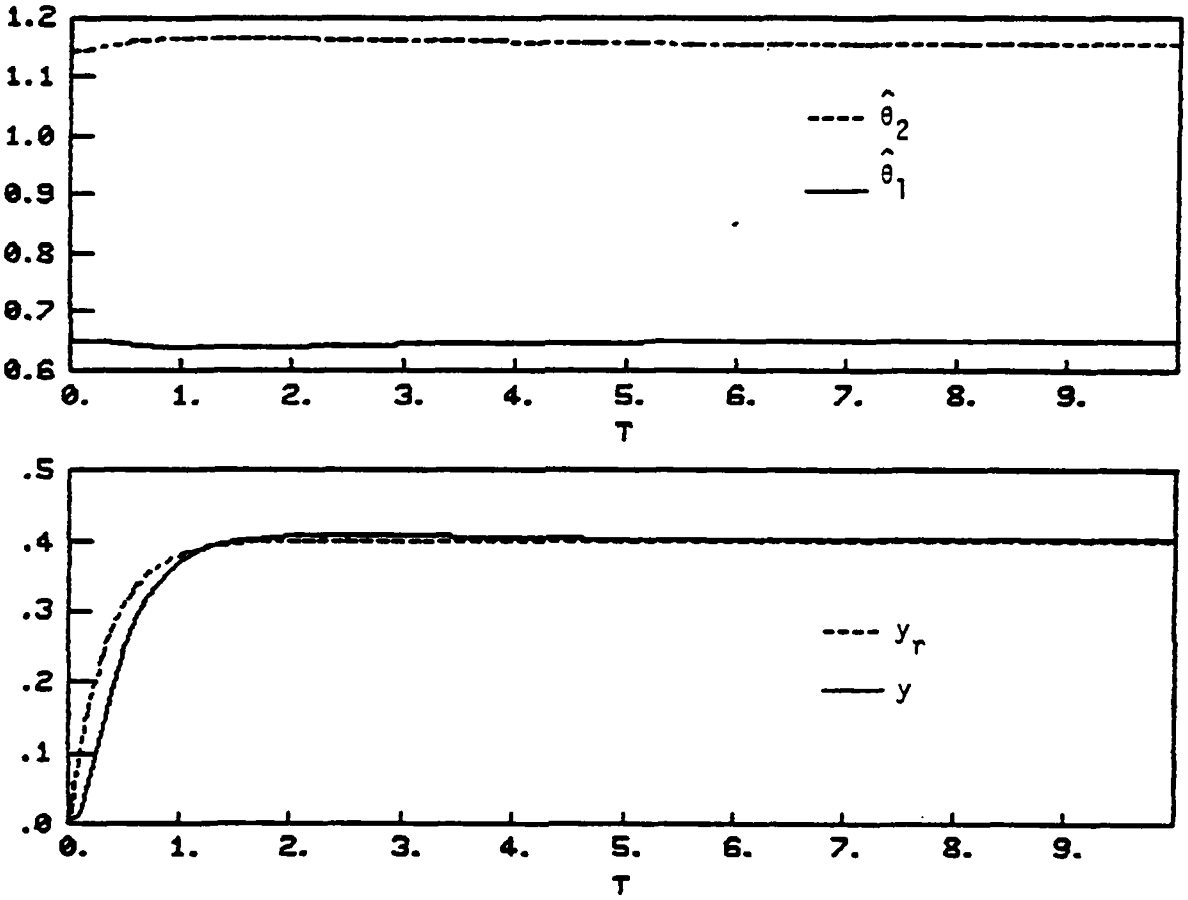

Figure 6.1 Response to r = .4

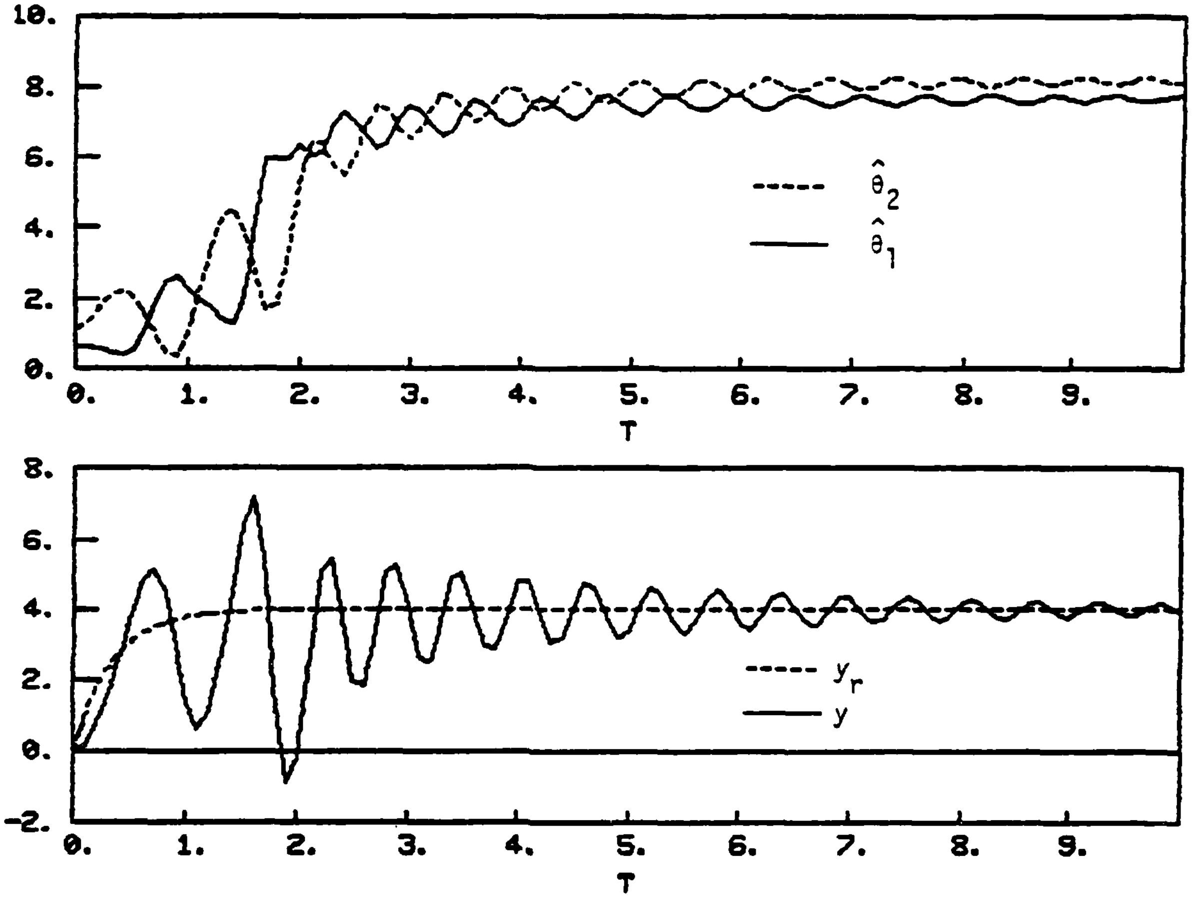

Figure 6.2 Response to r = 4.0

7. SPR COMPENSATION

In this section we heuristically develop a means to obtain global robust adaptive control. Since the SPR condition is violated whenever model error exceeds one, a natural scheme is to construct an SPR compensator which alleivates the problems by "filtering" the plant output; thus, avoiding the trouble. However, direct filtering does not change the size of model error. For example, with the plant $P = (1+\Delta)P_*$, let y_w denote the output of the filtered plant, where

$$y_w := Wy = Wd + (1+\Delta)WP_*u \tag{7.1}$$

Thus, model error is uneffected. Even filtering H_{ev} directly by W offers no help, since the bound (4.4c) is still less than one, i.e.,

$$|\tilde{H}_{ev}| < \mathrm{Re}(W\,\overline{H}_{ev})/|W\,\overline{H}_{ev}| < 1 \tag{7.2}$$

for any stable W. What we seek is an SPR compensator which only effects the unmodeled dynamics, but leaves the paramtric model intact.

A compensation scheme, which offers some promise as an SPR compensator, is that proposed in [22], as shown in Figure 7.1. To see the desired result suppose that $P = (1+\Delta)P_m$ with $\Delta \in B_S(\delta)$. Then, the compensator is equivalent to a plant which maps (u,d) into y_c where

$$y_c = Wd + P_c u \tag{7.2a}$$

$$\Delta_c := \frac{P_c - P_m}{P_m} \in B_S(W\delta) \tag{7.2b}$$

Thus, whenver $\delta(\omega) > 1$, select W(s) such that $|W(j\omega)|\delta(\omega) < 1$. The filter W acts like a "frequency switch" whose function is to insure condition (vi) of Lemma 5.1.

There are two ways to implement this compensator in an adaptive system. The first way is to use a fixed model of the plant for P_m, i.e., $P_m = \bar{P}$. The second way is to replace P_m with an adaptive observer, i.e., $P_m = \hat{P}$.

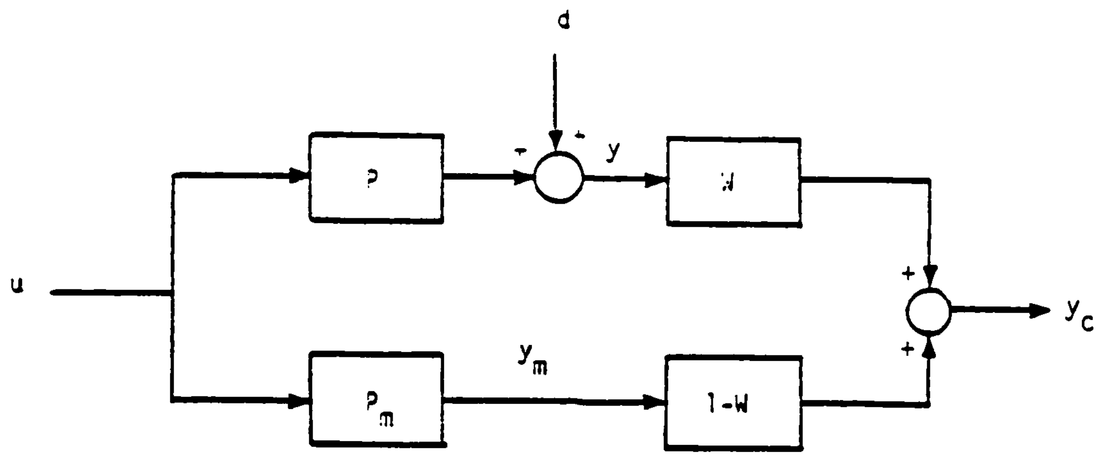

Figure 7.1 SPR Compensation

In either case, to obtain the benefit of the SPR compensator, the signal to be controlled is the compensator output y_c, not the plant output y. Both of these compensators will now be examined.

Fixed SPR Compensator

Let $P_m = \bar{P}$, a fixed model, and let the actual plant be given by (2.17), $P = (1+\Delta)P_*$ with $\Delta \in B_S(\delta)$. Then the fixed compensator plant equivalent model error (7.2b) is:

$$\Delta_c := \frac{P_c - P_*}{P_*} \in B_S(\delta_1) \tag{7.3a}$$

where

$$\delta_1(\omega) := |W(j\omega)|\delta(\omega) + |1 - W(j\omega)| \cdot \left|\frac{\bar{P}(j\omega) - P_*(j\omega)}{P_*(j\omega)}\right| \tag{7.3b}$$

This scheme is motivated by the fact that at low frequencies the tuned parametric model P_* is close to P; thus δ is small and $W \approx 1$. At high frequencies δ is large but $(\bar{P} - P_*)/P_*$ is small, $W \approx 0$ and so δ_1 is small. Of course the compensator is limited if there is large model error at intermediate frequencies.

Example 2

Example 1 is modified to include a fixed SPR compsnator with $W(s) = 1/(s+1)$ and $\bar{P}(s) = 2/(s+1)$. Simulation results with the large step command (r=4) are shown in Figure 7.2. Comparing these to Figure 6.2, without compensation, it is readily verified that the instability tendencies are eliminated. Also, direct calculations reveal that $H_{ev} \in SPR_0$, thus global stability is insured.

Adaptive SPR Compensation

An adaptive SPR compensator, together with the adaptive controller, is shown in Figure 7.3. The adaptive controller is described by,

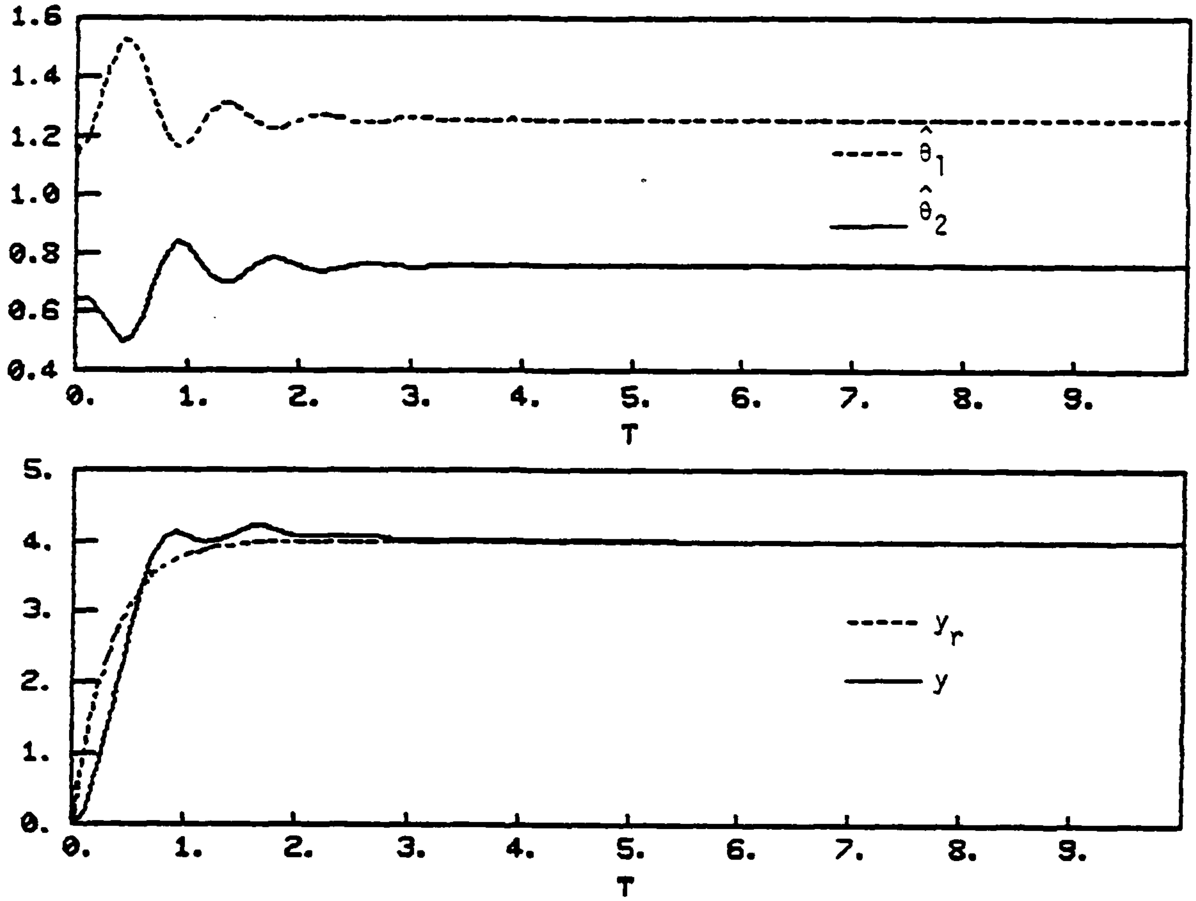

Figure 7.2 MRAC with SPR Compensator, r = 4.0

$$u = -\hat{\theta}_c' z_c \quad , \quad z_c' = \left(F_c' u \ , \ F_c'(y_c - r)\right) \tag{7.4a}$$

$$\dot{\hat{\theta}}_c = \Gamma_c \, z_c \, e_c \ , \quad e_c = y_c - y_r \tag{7.4b}$$

$$F_c'(s) = \left(1/L_c(s), \ldots, s^{n_c - 1}/L_c(s)\right) \ , \ n_c = \deg L_c(s) \tag{7.4c}$$

and the adaptive observer is described by,

$$\hat{y} = \hat{\theta}_o' \, z_o \quad , \quad z_o' = (F_o' \, u. \ -F_o' \, y) \tag{7.4d}$$

$$\dot{\hat{\theta}}_o = \Gamma_o z_o e_o \quad , \quad e_o = y - \hat{y} \tag{7.4d}$$

$$F_o'(s) = \left(1/L_o(s), \ \ldots, \ s^{n_o - 1}/L_o(s)\right) \ , \ n_o = \deg L_o(s) \tag{7.4f}$$

where $L_o(s)$ and $L_c(s)$ are both monic and stable. To generate the error system interconnection operators associated with this system, let θ_{*c} and θ_{*o} denote the tuned parameters with respective gain errors, θ_c and θ_o ; and let $v_c := \theta_c' z_c$ and $v_o := \theta_o' z_o$ be the corresponding adaptive control errors (3.6). By analogy with the procedure used in Section 5 we get,

$$u = C_*(r - y_c) - \frac{1}{1 + A_{*1}/L_c} v_c \tag{7.5}$$

$$\hat{y} = -\frac{B_{*1}}{L_o} d + \left(1 - \frac{B_{*1}}{L_o} \Delta\right) P_* u + v_o \tag{7.6}$$

where

$$C_* = \frac{A_{*2}/L_c}{1 + A_{*1}/L_c} \tag{7.7}$$

$$P_* = \frac{B_{*2}/L_o}{1 + B_{*1}/L_o} = \frac{gN_*}{D_*} \tag{7.8}$$

and where (A_{*1}, A_{*2}) are polynomials whose coefficients are the parameters in θ_{*c} ; (B_{*1}, B_{*2}) are polynomials whose coefficients are the parameters in θ_{*o} ; and N_*, P_* and g are as defined by (5.6a). The adaptive error model is given below in terms of T_*, S_*, and K_* as defined in (5.4). In additon,

define:

$$R := 1 + (W-1)\,\frac{D_*}{L_o} \tag{7.9}$$

The tuned signals are:

$$e_{*c} = S_*(1+\Delta RT_*)^{-1}R\;d + \left(T_*(1+\Delta R)(1+\Delta RT_*)^{-1} - H_r\right)r \tag{7.10a}$$

$$e_{*o} = D_*L_o^{-1}(1+\Delta RT_*)^{-1}d + D_*L_o^{-1}T_*\Delta(1+\Delta RT_*)^{-1}r \tag{7.10b}$$

$$z_{*c} = \begin{bmatrix} F_cA_{*2}L_c^{-1}P_*^{-1}K_*(1+\Delta RT_*)^{-1}(r-Rd) \\ \\ F_cS_*(1+\Delta RT_*)^{-1}(Rd-r) \end{bmatrix} \tag{7.10c}$$

$$z_{*o} = \begin{bmatrix} F_oA_{*2}L_c^{-1}P_*^{-1}K_*(1+\Delta RT_*)^{-1}(r-Rd) \\ \\ F_oT_*(1+\Delta RT_*)^{-1}\left(d - (1+\Delta)r\right) \end{bmatrix} \tag{7.10d}$$

The interconnections are:

$$H_{ev} = \begin{bmatrix} K_*(1+\Delta R)(1+\Delta RT_*)^{-1} & -(1-W)S_*(1+\Delta RT_*)^{-1} \\ \\ K_*D_*L_o^{-1}\Delta(1+\Delta RT_*)^{-1} & 1+(1-W)T_*D_*L_o^{-1}(1+\Delta RT_*)^{-1} \end{bmatrix} \tag{7.11a}$$

$$H_{z_cv} = \begin{bmatrix} F_cP_*^{-1}K_*(1+\Delta RT_*)^{-1} & F_cA_{*2}L_c^{-1}P_*^{-1}K_*(1-W)(1+\Delta RT_*)^{-1} \\ \\ F_cK_*(1+\Delta R)(1+\Delta RT_*)^{-1} & -F_cS_*(1-W)(1+\Delta RT_*)^{-1} \end{bmatrix} \tag{7.11b}$$

$$H_{z_0 v} = \begin{bmatrix} F_0 P_*^{-1} K_*(1+\Delta RT_*)^{-1} & F_0 A_{*2} L_c^{-1} P_*^{-1} K_*(1-W)(1+\Delta RT_*)^{-1} \\ -F_0 K_*(1+\Delta)(1+\Delta RT_*)^{-1} & -F_0 T_*(1-W)(1+\Delta)(1+\Delta RT_*)^{-1} \end{bmatrix} \tag{7.11c}$$

The factor $(1+\Delta RT_*)^{-1}$ appears in all the terms above. The transfer function R (7.9) reduces the effect of unmodeled dynamics; however not exactly by the amount anticipated, vis a vis (7.2). This is due to additonal model error introduced by the adaptive observer. Nonetheless, the model error attenuation is greater than with the fixed SPR compensator. In particular, at low frequencies $\Delta \approx 0$ and at high frequencies $R \approx 0$, since $W \approx 0$ and $D_* L_0^{-1} \approx 1$. Without further testing of H_{ev} (7.11a) it is not possible to state that $H_{ev} \in SPR_0$ at intermediate frequencies. Note, however, that the nominal value of H_{ev} is:

$$\bar{H}_{ev} = \begin{bmatrix} K_* & -(1-W)S_* \\ 0 & 1 \end{bmatrix} \tag{7.12}$$

which is SPR_0 provided that $K_* \in SPR$ and

$$\mathrm{Re}\, K_*(j\omega) > \frac{1}{4}|(1-W(j\omega))S_*(j\omega)|^2 \;, \; \omega \in R \tag{7.13}$$

Applying (4.11) to (7.11a), a tedious procedure, would give an upper bound on model error to insure $H_{ev} \in SPR_0$.

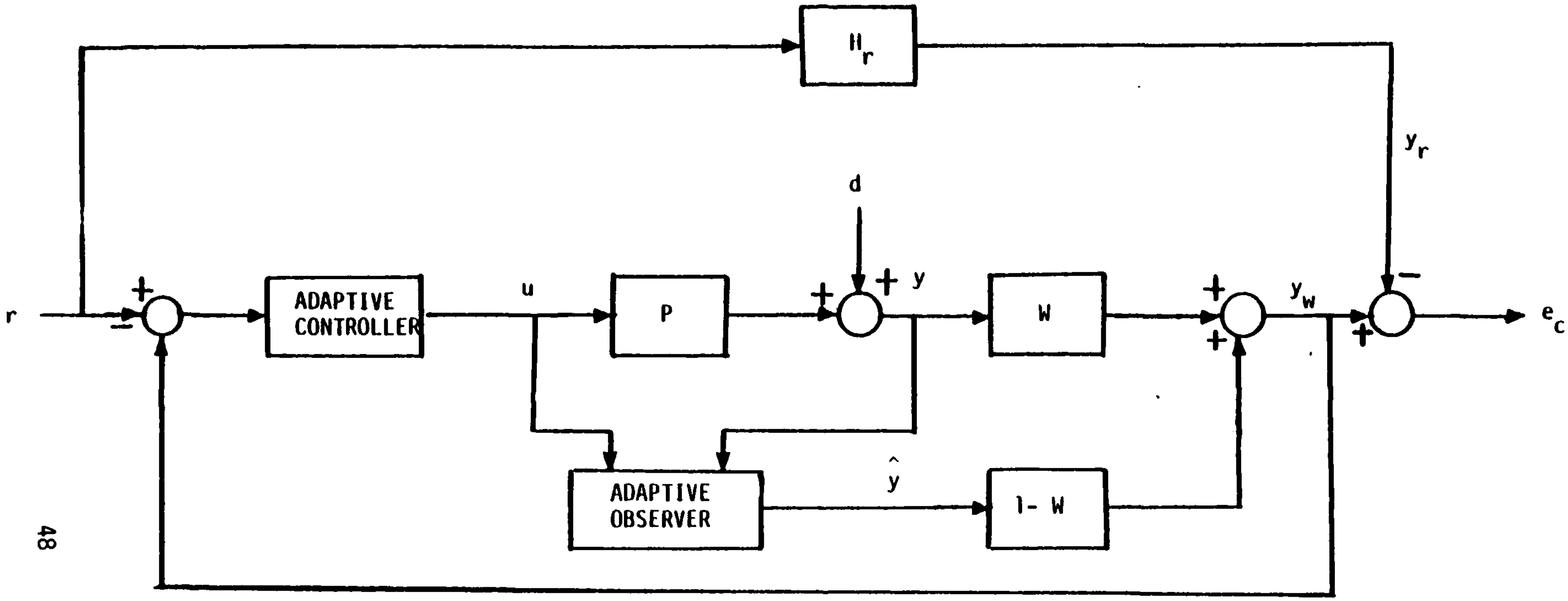

Figure 7.3 Adaptive SPR Compensator

8. CONCLUSIONS

This study has presented an input/output view of multivariable adaptive control for uncertain linear time invariant plants. The essence of the results are captured in Theorems 1A and 2B which provide conditions that guarantee global stability. Corollary 1 also give specific L_2 and L_∞ bounds on significant signals in the adaptive control system. These bounds, for example, can be used to guarantee that the adaptive system performs as well as a robust (non-adaptive) system using the same structure, but with fixed gains. By distinguishing between a tuned system and a robust system, we establish formulae which can be used to restrict the minimum performance improvement possible with the same control structure.

Although the stability results (Theorem 1A, 1B) are not entirely new (see e.g., [7],[8]), the input/output setting provides the means to directly determine the system robustness properties with respect to model error. The type of model error examined can arise from a variety of causes, such as unmodeled dynamics and reduced order modeling. It is very difficult to treat this type of "unstructured" dynamic model error by using Lyapunov theory, since the system order may not be known -- in fact, it may be infinite. Although infinite dimensional (distributed) systems were not considered here, Theorem 1 can be modified to include them, e.g., [26].

The structure of Theorems 1A and 1B require that a particular subsystem operator, denoted H_{ev}, is strictly positive real (SPR). This requirement is not unique to this presentation - passivity requirements, in one form or another, dominate proofs of global stability for practically all adaptive control systems, including recursive identification algorithms. Unfortunately, although $H_{ev} \in$ SPR is robust to model error (Lemma 4.1), the bound on the model error is too small to be of practical use. Even the most benign neglected dynamics violate the bound.

Although this study is concerned with continuous-time systems, the theorems carry over virtually intact to discrete-time systems. This is a direct consequence of the portable nature of the input/output view. However, there is an important issue unique to discrete-time systems: plant

uncertainty is critical to where performance is actually measured, which is in continuous-time, not at the sampled-data points. As a consequence, it may be necessary to map the discrete portions of the adaptive system (most likely the controller) into continuous-time, i.e., the L_2-gains of the discrete-time operators in the interconnection map, which are associated with the adaptive discrete-time controller, would be needed rather the discrete-time ℓ_2-gains .

Another area worth pursuing is the adaptive control of non-linear plants. The plant uncertainty description (2.11) does not exclude non-linear plants. Note that slowly drifting parameters in an otherwise perfectly known LTI plant could yield the same uncertainty description as a non-linear plant approximated by a parametric LTI model. All that is required is that there exists a (possibly) infinite dimensional LTI system which matches the input/output bel [illegible] of the plant for each possible input/output pair. Of course, if the [illegible] iear, then the tuned control is likely to be non-linear, which raises [illegible] interesting issues for further research.

One final remark: the stability results presented here, as well as other known results, provide <u>global stability.</u> This is achieved by requiring $H_{ev} \varepsilon$ SPR , a condition which is difficult to maintain in normal circumstances. On the other hand, this is a sufficient conditon; violation of which does not necessarily lead to instability. The simple example presented here in Figure 6.1-6.2, illustrates the point. Other examples of this phenomena abound, e.g., [12]. It would appear then, that a more valid approach to providing a system-theoretic setting for adaptive control is to develop <u>local</u> stability conditions, which, hopefully, do not require that $H_{ev} \varepsilon$ SPR . Preliminary results on local stability support this hope, e.g., [33], [34].

III. WORK IN PROGRESS

The ADCON concept involves many different issues, as can be seen from the earlier discussion and from [36]-[38]. So far we have addressed the problem of designing a controller for a single subsystem, when the rest of the system is fixed. This represents only one step in an iterative procedure in which each subsystem performs its own controller design. We are currently investigating extensions of the theory of robust control and adaptive control to the case of interconnected subsystems, in which local controllers are designed sequentially (iteratively) or simultaneously. A number of different information structures are being considered. It seems that by providing each subsystem with structural information in addition to an aggregate (reduced order) model of the rest of the systems, it is possible to obtain simpler design schemes.

We are also investigating the application of lattice structures to the adaptive control problem. Our earlier work in this area seemed to have generated a considerable amount of interest (cf. [41]-[46]). This class of algorithms is especially well suited for large scale problems of the type considered in this project.

APPENDIX A

PROOF OF THEOREMS 1 AND 2

Preliminaries

The main ingredient in the proof is to show stability by means of passivity. Although there are many variations on this theme, a general result is given by the following.

Theorem A.1 ([21], [35]

Consider the feedback system of Figure A.1 below with causal operators G_1 and G_2 .

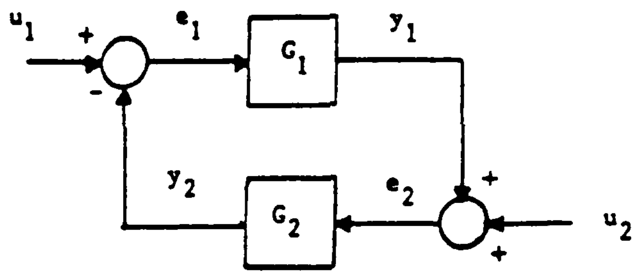

Figure A.1 Feedback System

Suppose there exists real constants ϵ_i, δ_i, α_i, $i=1,2$, such that

$$\langle x,G_i x\rangle_t > \epsilon_i \|x\|_{t2}^2 + \delta_i \|G_i x\|_{t2}^2 + \alpha_i, \ \forall t > 0, \ \forall x \in L_2[0,t] \qquad \text{(A.1)}$$

for $i=1,2$. Then the following holds $\forall t > 0$,

$$(\epsilon_2+\delta_1)\|y_1\|_{t2}^2 + (\epsilon_1+\delta_2)\|y_2\|_{t2}^2 < \|y_1\|_{t2}(\|u_1\|_{t2} + 2|\epsilon_2|\cdot\|u_2\|_{t2})$$

$$+ \|y_2\|_{t2}(\|u_2\|_{t2} + 2|\epsilon_1|\cdot\|u_1\|_{t2}) + |\epsilon_1|\cdot\|u_1\|_{t2}^2 + |\epsilon_2| \cdot \|u_2\|_{t2}^2$$

$$+ |\alpha_1| + |\alpha_2| \qquad \text{(A.2)}$$

Proofs of both theorems also rely on well known results for systems $H \varepsilon S_0^{nxm}$. The results required here are summarized in the following.

Theorem A-2 [see [19], Thm. 9, pg. 59]

Let $H \varepsilon S_0^{nxm}$; then:

(i) If $u \varepsilon L_2^m$, then $y = Hu \varepsilon L_2^n \quad L_\infty^n$, $\dot{y} \varepsilon L_2^n$, y is continuous, and $y(t) \to 0$ as $t \to \infty$.

(ii) If $u \varepsilon L_\infty^m$, then $y = Hu \varepsilon L_\infty^n$, $\dot{y} \varepsilon L_\infty^n$, and y is uniformly continuous.

(iii) If $u \varepsilon L_\infty^m$ and $u(t) \to$ constant $c \varepsilon R^m$ as $t \to \infty$, then $y(t) \to H(0)c$ exponentially as $t \to \infty$.

In order to simplify notation we drop the superstrict on L_p^n which indicates vector size.

We will establish Theorem 1A first. Some of the steps will be repeated for 1B. Also, without loss of generality, the matrix Γ in the adaptation law (3.15),(3.16) is set to identity. Corollary 1 is established as a by-product.

Proof of Theorem 1A

Part (i)

Identify G_1, G_2 in Figure A.1 with $e \to v$ and H_{ev} respectively. Also, let $u_1 = e_*$, $u_2 = 0$, $e_1 = e$, $y_1 = e_2 = v$, and $y_2 = H_{ev}v$. . Using adaptive law (3.15) we obtain,

$$\langle e,v\rangle_T = \langle e,Z'\theta\rangle_T = \langle Ze,\theta\rangle_T = \langle\dot{\theta}, \theta\rangle_T \tag{A.4}$$

$$= \frac{1}{2}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 \tag{A.5}$$

$$> -\frac{1}{2}\|\theta(0)\|^2 \tag{A.6}$$

Thus, using (A.1) gives,

$$\varepsilon_1 = \delta_1 = 0, \ \alpha_1 = -\frac{1}{2}\|\theta(0)\|^2 \tag{A.7}$$

Since $G_2 = H_{ev} \varepsilon_2 SPR_+$ by assumption, $\exists \mu, \gamma > 0$ such that $\forall\ x \varepsilon L_{2e}$, $\langle x, H_{ev}x\rangle_T > \mu\|x\|^2_{T2}$, $\|H_{ev}x\|_{T2} < \gamma\|x\|_{T2}$. Hence, from (A.1),

$$\varepsilon_2 = \mu, \ \delta_2 = \alpha_2 = 0 \tag{A.8}$$

Using Lemma A.1, together with (A.4)-(A.8) gives,

$$\|v\|_{T2} < \frac{1}{2\mu}[\|e_*\|_{T2} + (\|e_*\|^2_{T2} + 2\mu|\theta(0)|^2)^{1/2}] \tag{A.9}$$

$$\|e-e_*\|_{T2} < \gamma\|v\|_{T2} \tag{A.10}$$

$$|\theta(T)|^2 < |\theta(0)|^2 + 2\|e\|_{T2}\ \|v\|_{T2} \tag{A.11}$$

The bounds shown in (4.8) follow using the assumption $e_* \varepsilon L_2$. Hence, $e, v \varepsilon L_2$ and $\theta \varepsilon L_\infty$.

Having established that $v \varepsilon L_2$, Theorem A-2 $\Longrightarrow$ $\tilde{z} := z - z_* \ \varepsilon\ L_2 \cap L_\infty$, $\dot{\tilde{z}} \ \varepsilon\ L_2$, $\tilde{z} \to 0$, and $\tilde{z}$ is continuous. Since z_*, $\dot{z}_* \ \varepsilon\ L_\infty$ by assumption, it follows that $z \ \varepsilon\ L_\infty$ and $\dot{z} \ \varepsilon\ L_\infty$ ($\Longrightarrow$ z is uniformly continuous). Using $v = Z'\theta$ with z, $\theta' \ \varepsilon\ L_\infty \Longrightarrow v \ \varepsilon\ L_\infty$. Using $e = e_* - H_{ev}v$ with $e_* \ \varepsilon\ L_\infty$ and $H_{ev} \ \varepsilon\ S$ (by assumption), and $v \ \varepsilon\ L_\infty \Longrightarrow e \ \varepsilon\ L_\infty$. Hence, $\dot{\theta} = Ze \ \varepsilon\ L_\infty \Longrightarrow \theta$ is uniformly continuous $\Longrightarrow$ $v = Z'\theta$ is uniformly continuous (since z is) $\Longrightarrow$ $v \to 0$ since $v \ \varepsilon\ L_2$ is established. Using $v \to 0 \Longrightarrow e - e_* \to 0$, and since $e_* \to 0$ by assumption, $e \to 0$. Furthermore, $v \to 0 \Longrightarrow \tilde{z} \to 0$ exp. and $\dot{\theta} = Ze = \tilde{Z}e + Z_*e \to 0$, because $\tilde{z}$ and $e \to 0$. Using $\dot{v} = \dot{Z}'\theta + Z'\dot{\theta}$ with $\dot{z}$, θ, $\dot{\theta} \ \varepsilon\ L_\infty \Longrightarrow \dot{v} \ \varepsilon\ L_\infty$. Hence, $\dot{e} = \dot{e}_* - H_{ev}\dot{v} \ \varepsilon\ L_\infty$, because $e_* \ \varepsilon\ L_\infty$ by assumption. Thus, $\ddot{\theta} = \dot{Z}e + Z\dot{e} \ \varepsilon\ L_\infty$. This establishes properties (i-a)-(i-d).

To show (i-e) consider (3.15) written as:

$$\dot{\Theta} = -Z_* H_{ev} Z_*' \theta + w$$

$$w := -(Z_* H_{ev} \tilde{Z}' + \tilde{Z} H_{ev} Z_*' + \tilde{Z} H_{ev} \tilde{Z}')\theta \tag{A.12}$$

Since we have already established that $\tilde{z} \to 0$ exp. and $\theta \in L_\infty$, it follows that $w \to 0$ exp. Since $z_* \in$ PE by assumption (provided $e_* = 0$), $w \mapsto \theta$ is exp. stable by (2.15). Hence, $\theta \to 0$ exp. $\Rightarrow \dot{\theta}$, $v \to 0$ exp. $\Rightarrow e - e_* \to 0$ exp. This completes the proof of part (i) with adaptive law (3.15).

To show that (i-a)-(i-a) hold with adaptive law (3.16) requires showing that $G_1 : e \to v$ is passive. Consider the typical time interval,

$$I = \begin{cases} I_1 = \{t \in [t_0, t_1] \mid \|\hat{\theta}(t)\| < c\} \\ I_2 = \{t \in [t_1, t_2] \mid \|\hat{\theta}(t)\| > c > \max\|\theta_*\|\} \end{cases} \tag{A.13}$$

Hence,

$$\langle e,v\rangle_I = \langle e;v\rangle_{I_1} + \langle e,v\rangle_{I_2} \tag{A.14}$$

Thus,

$$\langle e,v\rangle_{I_1} = \langle \dot{\theta}, \theta\rangle_{I_1} = \frac{1}{2}\|\theta(t_1)\|^2 - \frac{1}{2}\|\theta(t_0)\|^2 \tag{A.15}$$

$$\langle e,v\rangle_{I_2} = \langle \dot{\theta} + (1 - \|\hat{\theta}\|/c)^2 \hat{\theta}, \theta\rangle_{I_2} \tag{A.16}$$

$$= \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_1)\|^2 + (1 - \|\hat{\theta}\|/c)^2 \langle\hat{\theta}, \theta\rangle_{I_2} \tag{A.17}$$

$$> \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_1)\|^2 \tag{A.18}$$

because $\langle\hat{\theta}, \theta\rangle_{I_2} > 0$ from,

$$\hat{\theta}(t)'\ \theta(t) = \hat{\theta}(t)'[\hat{\theta}(t)-\theta_*]$$
$$= \|\hat{\theta}(t)\|^2 - \hat{\theta}(t)'\theta_*$$
$$> \|\hat{\theta}(t)\|^2 - \|\hat{\theta}(t)\|c$$
$$= \|\hat{\theta}(t)\|(\|\hat{\theta}(t)\| - c) > 0, \qquad \forall t \in I_2 . \tag{A.19}$$

Thus,

$$\langle e,v\rangle_I > \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_0)\|^2 \tag{A.20}$$

Repeating the above procedure recursively,we eventually conclude that $\langle e,v\rangle_T > -\frac{1}{2}\|\theta(0)\|^2$ as before (A.6), and hence, $G_1 e \mapsto v$ is passive. The results in (i) now repeat for adaptive law (3.16). This completes the proof of part(i).

Proof of Theorem 1A,Part (ii)

Theorem 1A, Part (ii) is essentially an L_∞-stability result. The method of proof requires the notion of "exponential weighting" which is a means to obtain L_∞-stability of a system from the L_2-stability of an exponentially weighted version of the system (see e.g., [19], Chapter 9). We require the following:

Definition: Given a real number α define the exponential weighting operator by

$$x^\alpha(t) := \varepsilon^{\alpha t}x(t) \tag{A.21}$$

Consider the system y = Gu. An exp. weighted version of this system is denoted by $y^\alpha := G^\alpha u^\alpha$. Note that if G is a convolution operator with transfer function G(s) then G^α is also a convolution operator with transfer function $G(s-\alpha)$. Thus, the corresponding exponentialy weighted error system corresponding is described by

$$e^{\alpha} = e_{*}^{\alpha} - H_{ev}^{\alpha}\, v^{\alpha}$$

$$z^{\alpha} = z_{*}^{\alpha} - H_{zv}^{\alpha}\, v^{\alpha} \tag{A.22}$$

e

$$v^{\alpha} = Z'\theta^{\alpha}$$

$$\dot{\theta}^{\alpha} = \alpha\theta^{\alpha} + Ze^{\alpha} - \rho(\hat{\theta})\hat{\theta}^{\alpha}$$

where $\alpha > 0$ such that

$$H_{ev}^{\alpha} \in SPR_{+}^{m} \text{ and } H_{zv}^{\alpha} \in S_{0}^{k\times m} \tag{A.23}$$

Using Theorem A-1, identify G_1 with $e^{\alpha} \to v^{\alpha}$ and G_2 with H_{ev}^{α}. Note that it is always possible to find some $\alpha > 0$ such that (A.23) holds. We now examine the passivity of G_1: $e^{\alpha} \to v^{\alpha}$. Thus,

$$\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &= \langle e^{\alpha}, Z'\theta^{\alpha}\rangle_T = \langle Ze^{\alpha}, \theta^{\alpha}\rangle_T \\
&= \langle \theta^{\alpha}, \dot{\theta}^{\alpha} - \alpha\theta^{\alpha} + \rho(\hat{\theta})\hat{\theta}^{\alpha}\rangle_T \\
&= \frac{1}{2}\,\varepsilon^{2\alpha T}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 + \langle \rho(\hat{\theta})\hat{\theta}^{\alpha}, \theta^{\alpha}\rangle_T - \alpha\|\theta^{\alpha}\|_{T2}^2 \\
&> \frac{1}{2}\,\varepsilon^{2\alpha T}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 - \alpha\|\theta^{\alpha}\|_{T2}^2
\end{aligned} \tag{A.24}$$

The last line follows from (A.19), hence, (A.24) holds with or without the retardation term in the adaptive law. At this point there are two possibilities: either $\theta \in L_{\infty}$ or $|\theta(t)| \to \infty$ as $t \to \infty$. If $\theta \in L_{\infty}$ then $\exists$ constant $c_0 < \infty$ such that $\|\theta\|_{\infty} < c_0$. Then,

$$\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &> \frac{1}{2}\,\varepsilon^{2\alpha T}(\|\theta(T)\|^2\; c_0^2) - \frac{1}{2}\|\theta(o)\|^2 \\
&> -\frac{1}{2}\,\varepsilon^{2\alpha T}c_0^2 - \frac{1}{2}\|\theta(0)\|^2
\end{aligned} \tag{A.25}$$

If $|\theta(t)| \to \infty$ as $t \to \infty$ then it is always possible to select an arbitrarily large T such that $\|\theta(T)\| = \|\theta\|_{T\infty}$. Hence, for this T, (A.24) becomes,

$$\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &> \frac{1}{2}\,\varepsilon^{2\alpha T}(\|\theta(T)\|^2 - \|\theta\|_{T\infty}^2) - \frac{1}{2}\|\theta(0)\|^2 \\
&= -\frac{1}{2}\|\theta(0)\|^2
\end{aligned} \tag{A.26}$$

Thus, for some arbitrarily large T, (A.25) and (A.26) have the general form, i.e.,

$$\langle e^{\alpha}, v^{\alpha}\rangle_T \geq -c_1\, \varepsilon^{2\alpha T} - c_2 := -c(\alpha T) \tag{A.27}$$

where c_1, c_2 are non-negative constants. Hence,

$$\varepsilon_1 = \delta_1, = 0,\ \alpha_1 = -c(\alpha T) \tag{A.28}$$

Since $G_2 = H^{\alpha}_{ev} \varepsilon\ SPR_+$, $\exists$ constants μ, $\gamma > 0$ such that

$$\begin{aligned} &\langle x, H^{\alpha}_{ev}\, x\rangle_T \geq \mu \| x \|^2_{T2} \\ &\| H^{\alpha}_{ev}\, x \|_{T2} \leq \gamma \| x \|_{T2} \end{aligned} \tag{A.29}$$

Then,

$$\varepsilon_2 = \mu,\ \delta_2 = \alpha_2 = 0 \tag{A.30}$$

Using (A.2), we get

$$\| v^{\alpha} \|_{T2} \leq \frac{1}{2\mu}\left[\| e^{\alpha}_* \|_{T2} + \left(\| e^{\alpha}_* \|^2_{T2} + 2\mu\ c(\alpha T)\right)^{1/2}\right] \tag{A.31}$$

Since $e_* \varepsilon L_\infty$ by assumption,

$$\| e^{\alpha}_* \|_{T2} \leq \varepsilon^{\alpha T}(2\alpha)^{-1/2} \| e_* \|_\infty \tag{A.32}$$

Thus,

$$\| v^{\alpha} \|_{T2} \leq \frac{\varepsilon^{\alpha T}(2\alpha)^{-1/2}}{2\mu}\left[\| e_* \|_\infty + \left(\| e_* \|^2_\infty + 4\alpha\ \varepsilon^{-2\alpha T} \mu c(\alpha T)\right)^{1/2}\right] \tag{A.33}$$

Since $H^{\alpha}_{zv} \varepsilon\ S_0^{kxm}$, we obtain

$$|\tilde{z}(T)| = \left| \int_0^T H_{zv}(T-\tau) v(\tau) d\tau \right| \tag{A.34}$$

$$= \left| e^{-\alpha T} \int_0^T H^{\alpha}_{zv}(T-\tau) v^{\alpha}(\tau) d\tau \right| \tag{A.35}$$

$$< e^{-\alpha T} \| H^{\alpha}_{zv}(\cdot) \|_1 \cdot \| v^{\alpha} \|_{T2} \tag{A.36}$$

where $H^{\alpha}_{zv}(t)$ is the impulse response matrix associated with H^{α}_{zv}. Substituting (A.33) and (A.27) into (A.36) and noting that $e^{-2\alpha T} c(\alpha T) < c_1 + c_2$, we obtain,

$$|\bar{z}(T)| < \frac{(2\alpha)^{-1/2}}{\mu} \| H^{\alpha}_{zv}(\cdot) \|_1 \cdot [\| e_* \|_\infty + (\| e_* \|_\infty^2 + 4\alpha\mu(c_1+c_2))^{1/2}] \tag{A.37}$$

Since the right hand side is independent of T, and since T can be selected to be arbitrarily large, it follows that $z \in L_\infty$. Assuming there is no retardation or persistent excitation, this completes the proof of (ii-a) to (ii-d).

Assume now that $z \in PE$, which is a noncontradictory assumption since we have already shown that $z \in L_\infty$. Hence,

$$\dot{\theta} = -Z\, H_{ev}\, Z'\, \theta + Ze_* \tag{A.38}$$

Since $z \in PE$, $H_{ev} \in SPR_+$ and z, $\varepsilon_* \in L_\infty$, it follows from (2.15) that $(Ze_*, \theta(0)) \mapsto \theta$ is exp. stable, thus, $\theta, \dot{\theta} \in L_\infty$. The remaining results in (ii-e) follow immediately.

Suppose now that the adaptive law is given by (3.16). Then, we can write,

$$\begin{aligned} \dot{\hat{\theta}} &= Z\, e - \rho(\hat{\theta})\hat{\theta} = Z[e_* - H_{ev} Z'(\hat{\theta} - \theta_*)] - \rho(\hat{\theta})\hat{\theta} \\ &= w - Z\, H_{ev} Z'\, \hat{\theta} - \rho(\hat{\theta})\hat{\theta} \end{aligned} \tag{A.39}$$

where $w := Z\, e_* + Z\, H_{ev} Z'\theta_* \in L_\infty$, because z, $e_* \in L_\infty$. Consider the candidate Lyapunov function $V : t \mapsto \| \hat{\theta}(t) \|^2$. Hence,

$$\dot{V} = 2\, w'\hat{\theta} - \hat{\theta}'\, Z\, H_{ev} Z'\hat{\theta} - \rho(\hat{\theta})V \tag{A.40}$$

Suppose $\| \hat{\theta}(t) \| \to \infty$ as $t \to \infty$. Then there exists a time $T > 0$ such that

$\|\hat{\theta}(T)\| = \|\hat{\theta}\|_{T\infty} = V_T^{1/2} > c$. Hence,

$$\dot{V}_T < 2\|w\|_\infty V_T^{1/2} + \|z\|_\infty^2 \gamma_\infty(H_{ev})V_T - (1 - V_T^{1/2}/c)^2 V_T \qquad \text{(A.41)}$$

Clearly, there exists a finite constant c_1 such that when $V_T > c_1$, $\dot{V}_T < 0$. Therefore, $\hat{\theta}$ can not grow beyond all bounds, and hence, $\hat{\theta} \in L_\infty$. So then is θ and $\dot{\theta}$, and again the result of (ii-e) follow. This completes the proof of Theorem 1A. Note that in this case we do not obtain specific bounds on e, because the proof proceeds by contradition.

Proof of Theorem 1B

Part (i)

Since $H_{ev} \in SPR_0$, there exists $q > 0$ such that $G_{ev} := (1 + qs)H_{ev} \in SPR_+$, and furthermore, $G_{ev}^{-1} \in S$. As a result we can write (3.14a) as,

$$e = -H_{ev}\, y,\ y = v - G_{ev}^{-1}(e_* + q\, \dot{e}_*) \qquad \text{(A.42)}$$

Referring to Lemma A-1, let $G_1 : v \mapsto e$, $G_2 = H_{ev}$, $u_1 = 0$, and $u_2 = -G_{ev}^{-1}(e_* + q\dot{e}_*)$. Using (A.2) together with (A.42) and the passivity properties of H_{ev} gives,

$$\|e\|_{T2} < \frac{1}{2\mu}[\|u_2\|_{T2} + (\|u_2\|_{T2}^2 + 2\mu|\theta(0)|^2)^{1/2}] \qquad \text{(A.43)}$$

$$|\theta(T)| < |\theta(0)| + 2\|e\|_{T2} \cdot \|u_2\|_{T2} \qquad \text{(A.44)}$$

where μ is defined in (4.9a). Using (4.9b) gives, $\|u_2\|_{T2} < (1/k)\|e_* + q\dot{e}_*\|_{T2}$. This together with (A.43), (A.44) and the assumption $e_*, \dot{e}_* \in L_2$ gives the bounds shown in (4.9). Hence, $e \in L_2$, $\theta \in L_\infty$. However, we can not conclude that $v \in L_2$ as in Theorem 1A, part (i). From (A.42), we can conclude that $(1 + qs)^{-1} v \in L_2$. Since $G_{zv} := (1 + qs)H_{zv} \in S_{0_2}$, it follows from Lemma A-2 that $\tilde{z} := z - z_* \in L_2 \cap L_\infty$, $z \in L_2$ and $\tilde{z} \to 0$. Repeated use of Lemma A-2 and the error equations (3.14) gives the results (i-a) - (i-d). (i-e) follows from the arguments in the proof of Theorem 1A, part (i).

Part (ii)

The proof is entirely analgous to that of Theorem 1A, part (ii), where again we use exponential weighting.

APPENDIX B

PROOF OF LEMMA 5.1

The proof utilizes the following known results:

Definition: Let J denote a subset of S, consisting of functions in S whose inverse is also in S.

Fact [29]: If G is any scalar transfer function in R(s), then G has a coprime factorization in S, i.e., there exists N, D, A, and B in S such that $G = N/D$ and $AN + BD = 1$.

Lemma B-1: Consider the tuned adaptive system of Figure 5.2. Let $P_* \in R_o(s)$ and $C_* \in R_o(s)$ have coprime factorizations in S given by $P_* = N_p/D_p$ and $C_* = N_c/D_c$, respectively. Then, the elements of the transfer matrix from (r,d) into (e_*, z_*, y, u) all belong to S, if:

(i) $Q := D_pD_c + N_pN_c \in J$, (from [29]) (B.1)

and

(ii) $\delta(\omega)|T_*(j\omega)| < 1, \ \forall \ \omega \in R$, (from [16])

where

$$T_* := N_pN_c/Q := P_*C_*(1+P_*C_*)^{-1} \tag{B.2}$$

Using the definition of Q we can write H_{ev} and H_{zv} from (5.5) as,

$$H_{ev} = N_pQ^{-1}(1+\Delta)(1+\Delta T_*)^{-1} \tag{B.3}$$

$$H_{zv} = \begin{bmatrix} FD_pQ^{-1}(1+\Delta T_*)^{-1} \\ FN_pQ^{-1}(1+\Delta)(1+\Delta T_*)^{-1} \end{bmatrix} \tag{B.4}$$

From the definition of K_* (5.4b), we also obtain

$$Q = N_p K_*^{-1} \tag{B.3}$$

<u>Proof of Lemma 5.1</u>

We first show that (i), (ii), and (iv) => $Q \varepsilon J$. Let $P_* = N_p/D_p$ be a coprime factorization of $P*$ such that rel deg $D_p(s) = 0$. Since (i) => rel deg $P_*(s) = 1$, it follows that rel deg $N_p(s) = 1$. Moreover, (iv) => rel deg $K_*(s) = 1$, and that $K_1(s)$ and $K_2(s)$ are stable. This, together with (ii) and (B.3) establishes that $Q \varepsilon J$.

$H_{zv} \varepsilon S_o$ follows immediately by inspection of (B.2), since: $F \varepsilon S_o$ by assumption; D_p, $N_p \varepsilon S$; $Q \varepsilon J$; $\Delta \varepsilon S$ by assumption (vi); and finally (vi) => (ii) of Lemma B-1 => $(1+\Delta T_*)^{-1} \varepsilon S$.

Conditions (iv) and (vi) => $H_{ev} \varepsilon SPR_o$. This follows from Lemma 4.1 by letting $\bar{H}_{ev} = K_*$ and letting $1 + \bar{H}_{ev} = (1+\Delta)(1+\Delta T_*)^{-1}$. Thus, (4.4a) is satisfied since $K_* \varepsilon SPR_o$ from (iv). Also, from (4.4b),

$$k(\omega) = |\bar{H}_{ev}(j\omega)| = |\Delta(j\omega)S_*(j\omega)[1-\Delta(j\omega)T_*(j\omega)]^{-1}| \tag{B.4}$$

$$< \frac{\delta(\omega)|S_*(j\omega)|}{1-\delta(\omega)|T_*(j\omega)|} < \bar{k}(\omega) = \eta(\omega) \tag{B.5}$$

The last inequality comes from conditions (vi) and the definition of $\bar{k}(\omega)$ from (4.4b).

The final step in the proof of Lemma 5.1 is to show that there are a sufficient number of parameters in θ_* to insure a solution exists. This is guaranteed by satisfaction of condition (v). To see this combine (B.3) with the definition of Q from (B.1) to get

$$Q := N_c N_p + D_p D_c = N_p K_*^{-1} \tag{B.6}$$

From (5.2), let $N_c = A_{*2}/L$ and $D_c = 1 + A_{*1}/L$ be a coprime factorization of $C*$, and let $N_p = g\, N*/L$ and $D_p = 1 + D*/L$ be a coprime factorization of $P*$, where P_* is as defined in (i). With $K*$ given by (iv), (B.6) becomes the polynomial equation,

$$A_{*1}\, K_1 D_* + A_{*2} K_1 N_* = L(K_2 N_* - K_1 D_*) \tag{B.7}$$

Since $\deg(K_2 N_*) = \deg(K_1 D_*)$ and K_1, K_2, N_*, and D_* are all monic, it follows that $\deg[L(K_2 N_* - K_1 D_*] = \deg(L) + \deg(K_1) + \deg(D_*) - 1$. Then, using known results on polynomial equations, e.g. [30], it can be shown that (v) implies that (B.7) has a solution (A_{*1}, A_{*2}) .

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